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ISBN: 978-9917-0-1468-3

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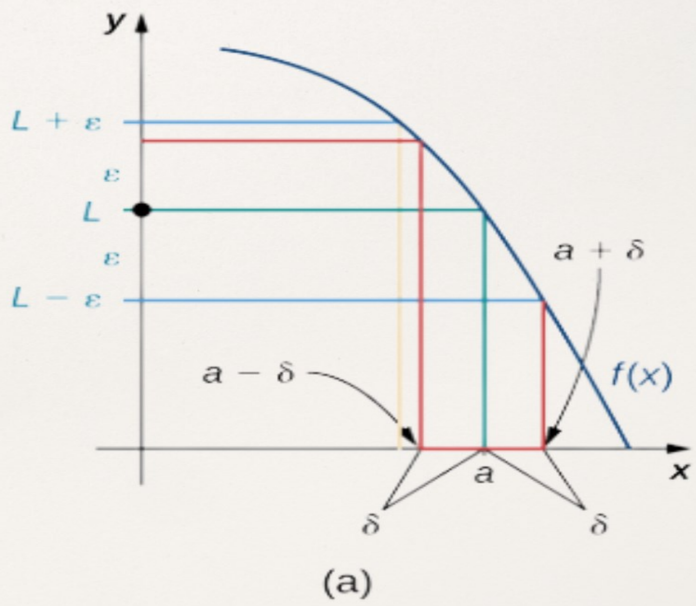
Límites Banco de Problemas Resueltos

LÍMITES

Banco de Problemas Resueltos



SUCRE – BOLIVIA

Lic. CPA Freddy A. Camargo Chambi

Límites

Banco de problemas resueltos

Freddy A. Camargo Chambi

2022

Título de la Obra

Limites Banco de Problemas Resueltos
Autor: Freddy Alejandro Camargo Chambi

Primera Edición
Año de Publicación: 2022

Depósito Legal: **3 – 1– 4 – 2022**

ISBN: **978 – 9917 - 0 – 1468 - 3**

Sucre - Bolivia

AGRADECIMIENTO

A "Dios" nuestro creador

A mi querida familia

Y a las personas que día a día se esfuerzan por ser personas de provecho y por colaborar para que Bolivia tenga un futuro mejor.

Presentación

Limites banco de problemas resueltos, es un libro para los estudiantes de la Facultad de Contaduría Pública y Ciencias Financieras, Carrera de Contaduría Pública para la Materia de Matemáticas (MAT 100). Se trata de un material pensando en los estudiantes de primer año de la carrera.

Cada ejercicio ha sido preparado en forma didáctica y entretenimiento presente, sin perder el rigor que exige la Matemática, será por tanto una herramienta para el estudiante.

Este Material se nutre con la experiencia de enseñanza del autor, como Auxiliar de docencia y capacitador en diferentes academias.

La pretensión de este libro es hacer conocer y entender los diferentes tipos de Limites y resolverlos de la forma más sencilla y accesible, sin perder lógicamente los márgenes de cientificidad y profundidad, que un trabajo de esta índole requiere; entonces, les corresponderá a los lectores juzgar en qué medida se alcanzó este objetivo.

Me queda, Agradecer al más grande matemático y calculador del universo, de quien sus obras combinan: razón, belleza, armonía y perfección, estoy hablando del creador "Dios" que mediante el, logremos descifrar los secretos del universo.

Sucre – Bolivia, diciembre de 2021

Lic. CPA Freddy A. Camargo Chambi

Experiencia del autor en la enseñanza

Freddy Alejandro Camargo Chambi ha culminado sus estudios en Contaduría Pública de la prestigiosa Facultad de Ciencias Económicas y Financieras de la UMSA (La Paz, Bolivia).

Se ha desempeñado como auxiliar de Docencia (Previo examen de competencia) en la ***Facultad de Ciencias Económicas y Financieras UMSA, de:***

- *GABINETE DE AUDITORIA FINANCIERA*
- *CONTABILIDAD INTERNACIONAL (2 Gestiones Consecutivas)*
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Como instructor en la Academia Newton*:*

- *MATEMÁTICA PREUNIVERSITARIA*
- *INTRODUCCIÓN A LA CONTABILIDAD*

Como instructor en la Academia La Paz

- *CÁLCULO I*
- *MATEMATICA FINANCIERA*
- *ESTADISTICA I y II*

Índice

““Infinito “no es expresión de una idea, sino un esfuerzo hacia ella. Representa un intento posible hacia una concepción imposible

El hombre necesitaba un término para indicar la dirección de este esfuerzo, la nube tras la cual se halla por siempre invisible, el objeto de esta tentativa

En fin, se requería una palabra por medio de la cual un hombre pudiera ponerse en relación, de inmediato con otro hombre y con cierta tendencia del intelecto humano.

De esta exigencia surgió la palabra “Infinito” la cual no representa, pues, sino el pensamiento de un pensamiento”

Edgar Alan Poe

Límites

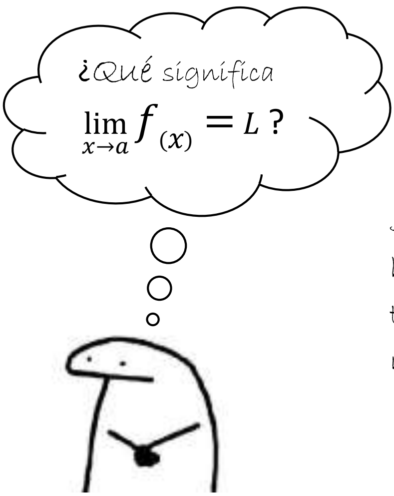
Definición de Límite

Sea $f_{(x)}$ la función definida sobre algún intervalo abierto que contiene el número a, excepto posiblemente en **a** misma. Entonces, decimos que el límite de $f_{(x)}$ cuando **x** tiene **a** es L, y lo expresamos como.

$$\lim_{x \to a} f_{(x)} = L$$

Sí para cada número $\varepsilon > 0$ Existe un número $\delta > 0$ tal que

Sí $0 < |x - a| < \delta$ entonces $\left|f_{(x)} - L\right| < \varepsilon$



Significa que los valores de $f_{(x)}$ pueden hacerse tan cercanos a **L** como queramos, tomando **x** lo suficientemente cerca de **a** (pero no igual a).

Teorema

El límite de $f_{(x)}$ existe y es único, cuando x tiende al valor de a, si y solo si existen los límites laterales y además son iguales.

$$\lim_{x\to a} f_{(x)} = L \Rightarrow \lim_{x\to a^-} f_{(x)} = \lim_{x\to a^+} f_{(x)} = L$$

Operaciones conocidas

$\infty + \infty = \infty$
$\infty \cdot \infty = \infty$
$\infty^{\infty} = \infty$
$\infty + a = \infty$
$\infty - a = \infty$
$\infty \cdot a = \infty$
$\frac{a}{\infty} = 0$
$\frac{0}{\infty} = 0$
$0^{\infty} = 0$

$\frac{\infty}{a} = \infty$
$\infty^{a} = \infty$
$a^{\infty} = \infty \ (a > 1)$
$a^{\infty} = 0 \ (a < 1)$
$\frac{\infty}{0} = \infty$
$\frac{0}{\infty} = 0$
$0^{\infty} = 0$

$$a \cdot 0 = 0$$
$$\frac{0}{a} = 0$$
$$\frac{a}{0} = \infty$$
$$0^a = 0$$
$$a^0 = 1$$

$$\ln 0 = -\infty$$
$$\ln 1 = 0$$
$$\log_a a = 1$$
$$\log \infty = \infty$$
$$\sin 0 = 0$$
$$\cos 0 = 1$$
$$\tan 0 = 0$$

Indeterminaciones

Son operaciones de resultados no conocidos, que adoptan un valor independiente de la función que les dio origen.

$$\frac{0}{0} =?$$
$$\frac{\infty}{\infty} =?$$
$$0 \cdot \infty =?$$
$$\infty^0 =?$$
$$\infty - \infty =?$$
$$1^\infty =?$$
$$0^0 =?$$

Al resolver ejercicios de límites, se reemplazará la variable por el valor al que tiende, cuando veamos que la función presenta **indeterminación**, esta debe ser levantada aplicando los diferentes métodos para su resolución

Propiedad de los límites

1	$\lim\limits_{x\to a} k = k \quad donde: \quad k = constante$
2	$\lim\limits_{x\to a} k \cdot f(x) = k \cdot \lim\limits_{x\to a} f(x)$
3	$\lim\limits_{x\to a} [f(x) \pm g(x)] = \lim\limits_{x\to a} f_{(x)} \pm \lim\limits_{x\to a} g(x)$
4	$\lim\limits_{x\to a} f(x) \cdot g(x) = \lim\limits_{x\to a} f(x) \cdot \lim\limits_{x\to a} g(x)$
5	$\lim\limits_{x\to a} \frac{f(x)}{g_{(x)}} = \frac{\lim\limits_{x\to a} f(x)}{\lim\limits_{x\to a} g_{(x)}} = \frac{L}{M}; \; M \neq 0$
6	$\lim\limits_{x\to a} \frac{k}{f(x)} = \frac{k}{\lim\limits_{x\to a} f(x)} = \frac{k}{m}; \; M \neq 0$
7	$\lim\limits_{x\to a} [f(x)]^n = \left[\lim\limits_{x\to a} f(x)\right]^n = \mathrm{L}^n \; ; \; n \in \mathbb{Z}^+$
8	$\lim\limits_{x\to a} \ln[f(x)] = \ln\left[\lim\limits_{x\to a} f(x)\right] = \ln L \; ; L > 0$

Limites

1.1 Limites Algebraicos

1.- $\lim_{x \to 3} \frac{x^3-2x-21}{x^4-27x}$

$$\lim_{x \to 3} \frac{3^3-2(3)-21}{3^4-27 \cdot 3} = \frac{0}{0}$$

Evaluar el limite antes de empezar a quitar la indeterminación

Factorizamos numerador

$$x^3 - 2x - 21 = (x-3) \cdot (x^2 + 3x + 27)$$

Factorizamos denominador

Recordamos productos notables

$$x^4 - 27x = x(x^3 - 27) = x(x^3 - 3^3)$$

$$= x(x-3)(x^2 + 3x + 3)$$

$$= \lim_{x \to 3} \frac{(x-3)(x^2+3x+7)}{x(x-3)(x^2+3x+9)} = \lim_{x \to 3} \frac{(x^2+3x+7)}{x(x^2+3x+9)}$$

Reemplazamos $x \to 3$

$$\lim_{x \to 3} \frac{(3^2 + 3 \cdot 3 + 7)}{3(3^2 + 3 \cdot 3 + 9)} = \frac{25}{81}$$

$$\lim_{x \to 3} \frac{x^3 - 2x - 21}{x^4 - 27x} = \frac{25}{81}$$

2.- $\lim_{x \to 2} \frac{x^3-x^2-8x+12}{x^3-x^2-12x+20}$

$$\lim_{x \to 2} \frac{2^3 - 2^2 - 8(2) + 12}{2^3 - 2^2 - 12(2) + 20} = \frac{0}{0}$$

Factorizamos numerador

$$x^3 - x^2 - 8x + 12 = (x+3)(x-2)(x-2)$$

Factorizamos denominador

$$x^3 - x^2 - 12x + 20 = (x-2)(x^2+x-10)$$

$$\lim_{x \to 2} \frac{(x-3)(x-2)(x-2)}{(x-2)(x^2+x-10)}$$

$$\lim_{x \to 2} \frac{(x-3)(x-2)}{(x^2+x-10)}$$

$$\lim_{x \to 2} \frac{(2-3)(2-2)}{(2^2+2-10)}$$

$$\frac{0}{a} = 0$$

$$\lim_{x \to 2} \frac{(-1)(0)}{(-4)} = 0$$

$$\lim_{x \to 2} \frac{x^3 - x^2 - 8x + 12}{x^3 - x^2 - 12x + 20} = 0$$

3.- $\lim_{x\to a}\frac{x^2-(a+1)x+a}{x^3-a^3}$

$$\lim_{x\to a}\frac{a^2-(a+1)a+a}{a^3-a^3}=\frac{a^2-(a+1)a+a}{a^3-a^3}=\frac{a^2-a^2-a+a}{a^3-a^3}=\frac{0}{0}$$

Factorizamos numerador

$$x^2-(a+1)x+a=(x-1)(x-a)$$

Factorizamos denominador

$$x^3-a^3=(x-a)(x^2+ax+a^2)$$

$$\lim_{x\to a}\frac{(x-1)(x-a)}{(x-a)(x^2+ax+a^2)}$$

$$\lim_{x\to a}\frac{(x-1)}{(x^2+ax+a^2)}$$

$$\lim_{x\to a}\frac{(a-1)}{(a^2+aa+a^2)}$$

$$\lim_{x\to a}\frac{(a-1)}{(a^2+a^2+a^2)}$$

$$\lim_{x\to a}\frac{(a-1)}{3a^2}$$

$$\lim_{x\to a}\frac{x^2-(a+1)x+a}{x^3-a^3}=\frac{(a-1)}{3a^2}$$

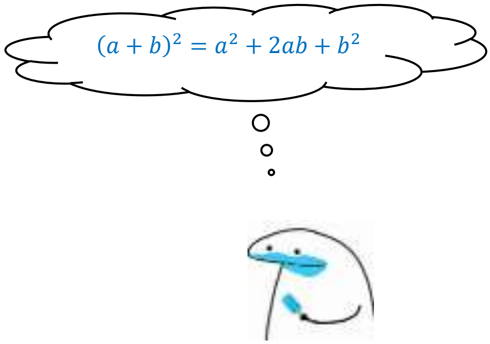
4.- $\lim_{x\to 1} \frac{1-x^2}{(1+ax)^2-(a+x)^2}, \qquad a > 0 \quad a \neq 1$

$$\lim_{x\to 1} \frac{1-1^2}{(1+a)^2-(a+1)^2} = \frac{0}{0}$$

$$\lim_{x\to 1} \frac{1-x^2}{1^2+2ax+a^2x^2-(a^2+2ax+x^2)}$$

$$\lim_{x\to 1} \frac{1-x^2}{1+2ax+a^2x^2-a^2-2ax-x^2}$$

$$\lim_{x\to 1} \frac{1-x^2}{1+a^2x^2-a^2-x^2}$$



$$\lim_{x\to 1} \frac{1-x^2}{1-x^2+a^2x^2-a^2}$$

$$\lim_{x\to 1} \frac{1-x^2}{1-x^2+a^2(x^2-1)}$$

$$\lim_{x\to 1} \frac{1-x^2}{(1-x^2)-a^2(1-x^2)}$$

$$\lim_{x\to 1} \frac{1-x^2}{(1-x^2)(1-a^2)}$$

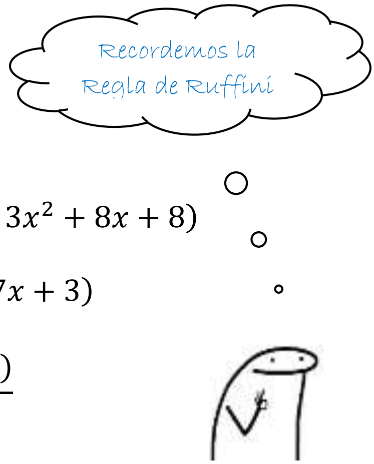
$$\lim_{x\to 1} \frac{1}{(1-a^2)}$$

$$\lim_{x\to 1} \frac{1-x^2}{(1+ax)^2-(a+x)^2} = \frac{1}{(1-a^2)}$$

5.- $\lim\limits_{x \to 1} \frac{5x^2+3x^5-8}{7x^4-4x-3}$

$$\lim_{x \to 1} \frac{5 \cdot 1^2 + 3 \cdot 1^5 - 8}{7 \cdot 1^4 - 4 \cdot 1 - 3} = \frac{8-8}{3-3} = \frac{0}{0}$$

$$\lim_{x \to 1} \frac{3x^5 + 5x^2 - 8}{7x^4 - 4x - 3}$$



$$3x^5 + 5x^2 - 8 = (x-1)(3x^4 + 3x^3 + 3x^2 + 8x + 8)$$

$$7x^4 - 4x - 3 = (x-1)(7x^3 + 7x^2 + 7x + 3)$$

$$\lim_{x \to 1} \frac{(x-1)(3x^4 + 3x^3 + 3x^2 + 8x + 8)}{(x-1)(7x^3 + 7x^2 + 7x + 3)}$$

$$\lim_{x \to 1} \frac{(3x^4 + 3x^3 + 3x^2 + 8x + 8)}{(7x^3 + 7x^2 + 7x + 3)}$$

$$\lim_{x \to 1} \frac{(3 \cdot 1^4 + 3 \cdot 1^3 + 3 \cdot 1^2 + 8 \cdot 1 + 8)}{(7 \cdot 1^3 + 7 \cdot 1^2 + 7 \cdot 1 + 3)}$$

$$\lim_{x \to 1} \frac{3+3+3+8+8}{7+7+7+3}$$

$$\lim_{x \to 1} \frac{25}{24}$$

$$\lim_{x \to 1} \frac{3x^5 + 5x^2 - 8}{7x^4 - 4x - 3} = \frac{25}{24}$$

6.- $\lim_{x\to 1}\frac{x^4+3x^3+7x^2-5x-6}{x^4+2x-3}$

$$\lim_{x\to 1}\frac{1^4+1^3+7\cdot 1^2-5\cdot 1-6}{1^4+2\cdot 1-3}=\frac{11-11}{3-3}=\frac{0}{0}$$

$$x^4+3x^3+7x^2-5x-6=(x-1)(x^3+4x^2+11x+6)$$

$$x^4+2x-3=(x-1)(x^3+x^2+x+3)$$

$$\lim_{x\to 1}\frac{(x-1)(x^3+4x^2+11x+6)}{(x-1)(x^3+x^2+x+3)}$$

$$\lim_{x\to 1}\frac{x^3+4x^2+11x+6}{x^3+x^2+x+3}$$

$$\lim_{x\to 1}\frac{1^3+4\cdot 1^2+11\cdot 1+6}{1^3+1^2+1+3}$$

$$\lim_{x\to 1}\frac{1+4+11+6}{1+1+1+3}$$

$$\lim_{x\to 1}\frac{22}{6}=\frac{11}{3}$$

$$\lim_{x\to 1}\frac{x^4+3x^3+7x^2-5x-6}{x^4+2x-3}=\frac{11}{3}$$

7.- $\lim_{x\to 0}\frac{(1+x)(1+2x)(2+3x)-1}{x}$

$$\lim_{x\to 0}\frac{(1+0)(1+2\cdot 0)(2+3\cdot 0)-1}{0}=\frac{0}{0}$$

Factorizamos el numerador

$$(1+x)(1+2x)(2+3x)=(1+3x+2x^2)(1+3x)$$

$$1+3x+2x^2+3x+9x^2+6x^3$$

$$1+6x+11x^2+6x^3$$

$$\lim_{x\to 0}\frac{1+6x+11x^2+6x^3-1}{x}$$

$$\lim_{x\to 0}\frac{6x+11x^2+6x^3}{x}$$

$$\lim_{x\to 0}\frac{x\cdot(6+11x+6x^2)}{x}$$

$$\lim_{x\to 0}\ 6+11x+6x^2$$

$$\lim_{x\to 0}\ 6+11\cdot 0+6\cdot 0^2$$

$$\lim_{x\to 0}\ 6$$

$$\lim_{x\to 0}\frac{(1+x)(1+2x)(2+3x)-1}{x}=6$$

$$8.-\lim_{x\to 0}\frac{(1+x)^5-(1+5x)}{x^2+x^5}$$

$$\lim_{x\to 0}\frac{(1+0)^5-(1+0\cdot 1)}{0^2+0^5}=\frac{0}{0}$$

Factorizamos el numerador

$$(1+x)^5=x^5+5x^4+10x^3+10x^2+5x+1$$

$$\lim_{x\to 0}\frac{x^5+5x^4+10x^3+10x^2+5x+1-(1+5x)}{x^2+x^5}$$

$$\lim_{x\to 0}\frac{x^5+5x^4+10x^3+10x^2+5x+1-1-5x}{x^2+x^5}$$

$$\lim_{x\to 0}\frac{x^5+5x^4+10x^3+10x^2}{x^2+x^5}$$

$$\lim_{x\to 0}\frac{x^2(x^3+5x^2+10x+10)}{x^2(1+x^3)}$$

$$\lim_{x\to 0}\frac{x^3+5x^2+10x+10}{1+x^3}$$

$$\lim_{x\to 0}\frac{0^3+5\cdot 0^2+10\cdot 0+10}{1+0^3}=10$$

$$\lim_{x\to 0}\frac{(1+x)^5-(1+5x)}{x^2+x^5}=10$$

9.-$\lim_{x\to 2}\left(\frac{2}{3x-6}-\frac{2}{2x^2-5x+2}\right)$

$$\lim_{x\to 2}\left(\frac{2}{3\cdot 2-6}-\frac{2}{2\cdot 2^2-5\cdot 2+2}\right)=\left(\frac{2}{6-6}-\frac{2}{10-10}\right)=\infty-\infty$$

$$\lim_{x\to 2}\frac{4x^2-10x+4-6x+12}{(2x^2-5x+2)(3x-6)}$$

$$\lim_{x\to 2}\frac{4x^2-16x+16}{(2x^2-5x+2)\cdot(3x-6)}$$

$$\lim_{x\to 2}\frac{4(x-2)^2}{(2x^2-5x+2)\cdot(3x-6)}$$

$$\lim_{x\to 2}\frac{4(x-2)^2}{(2x-1)(x-2)(3x-6)}$$

$$\lim_{x\to 2}\frac{4(x-2)^2}{(2x-1)(x-2)3(x-2)}$$

$$\lim_{x\to 2}\frac{4(x-2)^2}{3(2x-1)(x-2)^2}$$

$$\lim_{x\to 2}\ \frac{4}{3(2x-1)}$$

$$\lim_{x\to 2}\frac{4}{3(2\cdot 2-1)}=\frac{4}{3(3)}=\frac{4}{9}$$

$$\lim_{x\to 2}\left(\frac{2}{3x-6}-\frac{2}{2x^2-5x+2}\right)=\frac{4}{9}$$

10.- $\lim\limits_{x\to 1}\frac{x^{100}-2x+1}{x^{50}-2x+1}$

$$\lim_{x\to 1}\frac{1^{100}-2\cdot 1+1}{1^{50}-2\cdot 1+1}=\frac{2-2}{2-2}=\frac{0}{0}$$

$$\lim_{x\to 1}\frac{x^{100}-2x+1-1+1}{x^{50}-2x+1-1+1}$$

$$\lim_{x\to 1}\frac{x^{100}-1-2x+2}{x^{50}-1-2x+2}$$

$$\lim_{x\to 1}\frac{\frac{(x^{100}-1)-2(x-1)}{x-1}}{\frac{(x^{90}-2)-2(x-1)}{x-1}}=\lim_{x\to 1}\frac{\frac{x^{100}-1}{x-1}-2}{\frac{x^{50}-1}{x-1}-2}$$

$$\frac{a^n-b^n}{a-b}=a^n+a^{n-1}b+a^{n-2}b^2+\cdots+ab^{n-1}+b^n$$

$$\lim_{x\to 1}\frac{\frac{x^{100}-1}{x-1}-2}{\frac{x^{50}-1}{x-1}-2}=\lim_{x\to 1}\frac{(x^{99}+x^{98}+x^{97}+\cdots x+1)-2}{(x^{49}+x^{48}+x^{47}+\cdots x+1)-2}$$

$$\lim_{x\to 1}\frac{(1^{99}+1^{98}+1^{97}+\cdots 1+1)-2}{(1^{49}+1^{48}+1^{47}+\cdots 1+1)-2}=\frac{100-2}{50-2}=\frac{98}{48}=\frac{49}{24}$$

$$\lim_{x\to 1}\frac{x^{100}-2x+1}{x^{50}-2x+1}=\frac{49}{24}$$

11.- $\lim_{X\to 1}\frac{x^{\mathrm{m}}-1}{x^{n}-1}\ m,n\in\mathbb{Z}^{+}$

$$\lim_{X\to 1}\frac{1^{\mathrm{m}}-1}{1^{n}-1}=\frac{0}{0}$$

$$\lim_{X\to 1}\frac{x^{\mathrm{m}}-1}{x^{n}-1}=\lim_{X\to 1}\frac{\frac{x^{\mathrm{m}}-1}{x-1}}{\frac{x^{\mathrm{m}}-1}{x-1}}$$

$$\lim_{X\to 1}\frac{\frac{x^{\mathrm{m}}-1}{x-1}}{\frac{x^{\mathrm{n}}-1}{x-1}}=\lim_{X\to 1}\frac{(x^{m-1}+x^{\mathrm{m}-2}+x^{\mathrm{m}-3}+\cdots+1)}{(x^{n-1}+x^{\mathrm{n}-2}+x^{\mathrm{n}-3}+\cdots+1)}$$

$$\lim_{X\to 1}\frac{(1^{m-1}+1^{\mathrm{m}-2}+1^{\mathrm{m}-3}+\cdots+1)}{(1^{n-1}+1^{\mathrm{n}-2}+1^{\mathrm{n}-3}+\cdots+1)}=\frac{m}{n}$$

$$\lim_{X\to 1}\frac{x^{\mathrm{m}}-1}{x^{n}-1}=\frac{m}{n}$$

12.-$\lim_{x\to 1}\frac{x^{2m}-1}{x^{2}-1}$

$$\lim_{x\to 1}\frac{1^{2m}-1}{1^{2}-1}=\frac{0}{0}$$

$$\lim_{x\to 1}\frac{x^{2m}-1}{x^2-1}=\lim_{x\to 1}\frac{x^{2m}-1}{(x-1)(x+1)}$$

$$\lim_{x\to 1}\frac{x^{2m}-1}{(x-1)(x+1)}=\lim_{x\to 1}\frac{x^{2m}-1}{(x-1)}\cdot\frac{1}{(x+1)}$$

$$\lim_{x\to 1}(x^{2m-1}+x^{2\text{m}-2}+x^{2\text{m}-3}+\cdots+1)\cdot\frac{1}{(x+1)}$$

$$\lim_{x\to 1}(1^{2m-1}+1^{2\text{m}-2}+1^{2\text{m}-3}+\cdots+1)\cdot\frac{1}{(1+1)}$$

$$\lim_{x\to 1}\frac{2m}{2}=m$$

$$\lim_{x\to 1}\frac{x^{2m}-1}{x^2-1}=m$$

13.- $\lim_{x\to 1}\frac{x^{14}+x^2-2}{x^{12}+4x^8+x^2-6}$

$$\lim_{x\to 1}\frac{1^{14}+1^2-2}{1^{12}+4\cdot 1^8+1^2-6}=\frac{0}{0}$$

Cambio de Variable sea:

$$F=x^2 \qquad x\to 1\ \Rightarrow u\to 1$$

$$\lim_{x\to 1}\frac{(x^2)^7+x^2-2}{(x^2)^6+4(x^2)^4+x^2-6}$$

$$\lim_{x\to 1}\frac{(F)^7+\mathrm{F}-2}{(F)^6+4(F)^4+F-6}$$

$$\lim_{x\to 1}\frac{F^7-1+\mathrm{F}-1}{(F)^6-1+4(F)^4-4+F-1}$$

$$\lim_{x\to 1}\frac{(\mathrm{F}-1)(F^6+F^5+F^4+F^3+F^2+F+1)+(\mathrm{F}-1)}{F^6-1+4F^4-4+F-1}$$

$$\lim_{x\to 1}\frac{(\mathrm{F}-1)(F^6+F^5+F^4+F^3+F^2+F+1)+(\mathrm{F}-1)}{(\mathrm{F}-1)(F^5+F^4+F^3+F^2+F+1)+4(F^4-1)+(F-1)}$$

$$\lim_{x\to 1}\frac{(\mathrm{F}-1)(F^6+F^5+F^4+F^3+F^2+F+1)+(\mathrm{F}-1)}{(\mathrm{F}-1)(F^5+F^4+F^3+F^2+F+1)+4(F-1)(F^3+F^2+F+1)+(F-1)}$$

$$\lim_{x\to 1}\frac{(\mathrm{F}-1)[(F^6+F^5+F^4+F^3+F^2+F+1)+1]}{(\mathrm{F}-1)[(F^5+F^4+F^3+F^2+F+1)+4(F^3+F^2+F+1)+1]}$$

$$\lim_{x\to 1}\frac{F^6+F^5+F^4+F^3+F^2+F+1+1}{F^5+F^4+F^3+F^2+F+1+4F^3+4F^2+4F+4+1}$$

$$\lim_{x\to 1}\frac{1^6+1^5+1^4+1^3+1^2+1+1+1}{1^5+1^4+1^3+1^2+1+1+4\cdot 1^3+4\cdot 1^2+4\cdot 1+4+1}$$

$$\lim_{x\to 1}\frac{8}{23}$$

$$\lim_{x\to 1}\frac{x^{14}+x^2-2}{x^{12}+4x^8+x^2-6}=\frac{8}{23}$$

14. $\lim_{x\to 0}\frac{(3+x)^n-3^n}{x}$

$$\lim_{x\to 0}\frac{(3+0)^n-3^n}{0}=\frac{0}{0}$$

$$\lim_{x\to 0}\frac{3^n+\frac{n}{1!}\cdot 3^{n-1}\cdot x\frac{+11(n-1)}{2!}x^2+\cdots x^n-3^n}{x}$$

$$\lim_{x\to 0}\frac{\frac{n}{1!}3^{n-1}x+\frac{\mathrm{n}(n-1)}{2!}x^2+\cdots x^n}{x}$$

$$\lim_{x\to 0}\frac{x\left(\frac{n}{1!}3^{n-1}+\frac{\mathrm{n}(n-1)}{2!}x+\cdots x^{n-1}\right)}{x}$$

$$\lim_{x\to 0}\frac{n}{1!}3^{n-1}+\frac{\mathrm{n}(n-1)}{2!}x+\cdots x^{n-1}$$

$$\lim_{x\to 0} \frac{n}{1!}3^{n-1} + \frac{\text{n}(n-1)}{2!} \cdot 0 + \cdots + 0^{n-1}$$

$$\lim_{x\to 0} \frac{n}{1!}3^{n-1} = n3^{n-1}$$

$$\lim_{x\to 0} \frac{(3+x)^n - 3^n}{x} = n3^{n-1}$$

15.-$\lim_{x\to 1} \frac{(3x+5)^2-64}{x^2+x-2}$

$$\lim_{x\to 1} \frac{(3(1)+5)^2 - 64}{(1)^2 + (1) - 2} = \frac{0}{0}$$

$$\lim_{x\to 1} \frac{9x^2 + 30x + 25 - 64}{x^2 + x - 2}$$

$$\lim_{x\to 1} \frac{9x^2 + 30x - 39}{x^2 + x - 2}$$

$$\lim_{x\to 1} \frac{3(3x^2 + 10x - 13)}{(x+2)(x-1)}$$

$$\lim_{x\to 1} \frac{3(3x+13)(x-1)}{(x+2)(x-1)}$$

$$\lim_{x\to 1} \frac{3(3x+13)}{(x+2)} = \frac{3(3\cdot 1 + 13)}{(1+2)} = \frac{3(16)}{3}$$

$$\lim_{x\to 1}\frac{(3x+5)^2-64}{x^2+x-2}=16$$

16.- $\lim\limits_{x\to 0}\frac{(x+3)(x+2)-(x+1)(x+6)}{(x+2)(x+4)-(x+1)(x+8)}$

$$\lim_{x\to 0}\frac{(0+3)(0+2)-(0+1)(0+6)}{(0+2)(0+4)-(0+1)(0+8)}=\frac{6-6}{8-8}=\frac{0}{0}$$

$$\lim_{x\to 0}\frac{(x^2+5x+6)-(x^2+7x+6)}{(x^2+6x+8)-(x^2+9x+8)}$$

$$\lim_{x\to 0}\frac{x^2+5x+6-x^2-7x-6}{x^2+6x+8-x^2-9x-8}$$

$$\lim_{x\to 0}\frac{5x-7x}{6x-9x}$$

$$\lim_{x\to 0}\frac{-12x}{-3x}=4$$

$$\lim_{x\to 0}\frac{(x+3)(x+2)-(x+1)(x+6)}{(x+2)(x+4)-(x+1)(x+8)}=4$$

17.- $\lim\limits_{x\to -\frac{1}{2}}\left(\frac{x+1}{2x^2-3-2}+\frac{x}{2x^2+7x+3}\right)$

$$\lim_{x\to -\frac{1}{2}}\frac{x+1}{(2x+1)(x-2)}+\frac{x}{(x+3)(2x+1)}$$

$$\lim_{x\to-\frac{1}{2}} \frac{-\frac{1}{2}+1}{\left(2\cdot-\frac{1}{2}+1\right)\left(-\frac{1}{2}-2\right)} + \frac{-\frac{1}{2}}{\left(-\frac{1}{2}+3\right)\left(2\cdot-\frac{1}{2}+1\right)} = \infty - \infty$$

$$\lim_{x\to-\frac{1}{2}} \frac{x+1}{(2x+1)(x-2)} + \frac{x}{(x+3)(2x+1)}$$

$$\lim_{x\to-\frac{1}{2}} \frac{(x+1)(x+3)+x(2x+1)(x-2)}{(2x+1)(x-2)(x+3)}$$

$$\lim_{x\to-\frac{1}{2}} \frac{x^2+4x+3-x^2+2x}{(2x+1)(x-2)(x+3)}$$

$$\lim_{x\to-\frac{1}{2}} \frac{6x+3}{(2x+1)(x-2)(x+3)}$$

$$\lim_{x\to-\frac{1}{2}} \frac{3(2x+1)}{(2x+1)(x-2)(x+3)}$$

$$\lim_{x\to-\frac{1}{2}} \frac{3}{(x-2)(x+3)} = \frac{3}{\left(-\frac{1}{2}-2\right)\left(-\frac{1}{2}+3\right)} = \frac{3}{\left(-\frac{5}{2}\right)\left(\frac{5}{2}\right)}$$

$$\lim_{x\to-\frac{1}{2}} = -\frac{3}{\frac{25}{4}} = -\frac{12}{25}$$

$$\lim_{x\to-\frac{1}{2}} \left(\frac{x+1}{2x^2-3-2} + \frac{x}{2x^2+7x+3}\right) = -\frac{12}{25}$$

18.- $\lim_{x \to a} \frac{x^n - a^n - na^{n-1}(x-a)}{(x-1)^2}$

$$\lim_{x \to a} \frac{a^n - a^n - na^{n-1}(a-a)}{(a-1)^2}$$

$$\lim_{x \to a} \frac{0 - na^{n-1}(0)}{(a-1)^2}$$

$$\lim_{x \to a} \frac{0}{(a-1)^2} = 0$$

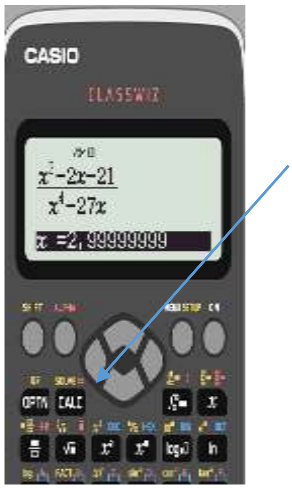
$$\lim_{x \to a} \frac{x^n - a^n - na^{n-1}(x-a)}{(x-1)^2} = 0$$

1.2 Calculo de Limites Algebraico con la Calculadora

$$19.-\lim_{x\to 3}\frac{x^3-2x-21}{x^4-27x}$$

$$\lim_{x\to 3}\frac{3^3-2(3)-21}{3^4-27\cdot 3}=\frac{0}{0}$$

Copiamos a la Calculadora



Presionamos la tecla CALC

Introducimos un número que se aproxime a 3 en este caso será 2,999999

Presionamos la tecla =

$$\lim_{x\to 3}\frac{x^3-2x-21}{x^4-27x}=\frac{25}{81}$$

20.-$\lim_{x\to 1}\frac{5x^2+3x^5-8}{7x^4-4x-3}$

$$\lim_{x\to 1}\frac{51^2+31^5-8}{71^4-41-3}=\frac{8-8}{3-3}=\frac{0}{0}$$

$$\lim_{x\to 1}\frac{3x^5+5x^2-8}{7x^4-4x-3}=\frac{25}{24}$$

21.- $\lim\limits_{x \to 1} \frac{x^4+3x^3+7x^2-5x-6}{x^4+2x-3}$

$$\lim_{x \to 1} \frac{1^4+1^3+7\cdot 1^2-5\cdot 1-6}{1^4+2\cdot 1-3} = \frac{11-11}{3-3} = \frac{0}{0}$$

$$\lim_{x \to 1} \frac{x^4+3x^3+7x^2-5x-6}{x^4+2x-3} = \frac{11}{3}$$

22.- $\lim\limits_{x \to 1} \frac{x^{100}-2x+1}{x^{50}-2x+1}$

$$\lim_{x \to 1} \frac{1^{100}-2\cdot 1+1}{1^{50}-2\cdot 1+1} = \frac{0}{0}$$

$$\lim_{x \to 1} \frac{x^{100}-2x+1}{x^{50}-2x+1} = \frac{49}{24}$$

23.-$\lim\limits_{x\to 1}\frac{x^{14}+x^2-2}{x^{12}+4x^8+x^2-6}$

$$\lim_{x\to 1}\frac{1^{14}+1^2-2}{1^{12}+4\cdot 1^8+1^2-6}=\frac{0}{0}$$

$$\lim_{x\to 1}\frac{x^{14}+x^2-2}{x^{12}+4x^8+x^2-6}=\frac{8}{23}$$

24.-$\lim\limits_{x\to 1}\frac{(3x+5)^2-64}{x^2+x-2}$

$$\lim_{x\to 1}\frac{(3(1)+5)^2-64}{(1)^2+(1)-2}=\frac{0}{0}$$

$$\lim_{x\to 1}\frac{(3x+5)^2-64}{x^2+x-2}=16$$

1.3 Limites con radicales

25. $-\lim_{x\to 0} \frac{\sqrt{1+x^2}-1}{x^2}$

$$\lim_{x\to 0} \frac{\sqrt{1+0^2}-1}{0^2} = \frac{0}{0}$$

$$\lim_{x\to 0} \frac{\sqrt{1+x^2}-1}{x^2} \cdot \frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}+1}$$

$$\lim_{x\to 0} \frac{(\sqrt{1+x^2})^2-1^2}{x^2} \cdot \frac{1}{\sqrt{1+x^2}+1}$$

$$\lim_{x\to 0} \frac{1+x^2-1}{x^2} \cdot \frac{1}{\sqrt{1+x^2}+1}$$

$$\lim_{x\to 0} \frac{x^2}{x^2} \cdot \frac{1}{\sqrt{1+x^2}+1}$$

$$\lim_{x\to 0} \frac{1}{\sqrt{1+x^2}+1} = \lim_{x\to 0} \frac{1}{\sqrt{1+0^2}+1} = \lim_{x\to 0} \frac{1}{1+1}$$

$$\lim_{x\to 0} \frac{1}{2}$$

$$\lim_{x\to 0} \frac{\sqrt{1+x^2}-1}{x^2} = \frac{1}{2}$$

26.-$\lim\limits_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}$

$$\lim_{x\to 0}\frac{\sqrt{1+0}-\sqrt{1+0}}{0}=\frac{0}{0}$$

$$\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}\cdot\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}$$

$$\lim_{x\to 0}\frac{\sqrt{1+x}^2-\sqrt{1-x}^2}{x}\cdot\frac{1}{\sqrt{1+x}+\sqrt{1-x}}$$

$$\lim_{x\to 0}\frac{1+x-(1-x)}{x}\cdot\frac{1}{\sqrt{1+x}+\sqrt{1-x}}$$

$$\lim_{x\to 0}\frac{1+x-1+x}{x}\cdot\frac{1}{\sqrt{1+x}+\sqrt{1+x}}$$

$$\lim_{x\to 0}\frac{2x}{x}\cdot\frac{1}{\sqrt{1+x}+\sqrt{1+x}}$$

$$\lim_{x\to 0}\frac{2}{\sqrt{1+x}+\sqrt{1+x}}=\lim_{x\to 0}\frac{2}{\sqrt{1+0}+\sqrt{1+0}}$$

$$\lim_{x\to 0}\frac{2}{1+1}=\frac{2}{2}=1$$

$$\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}=1$$

27.-$\lim_{x\to 5}\frac{\sqrt{x-4}-\sqrt{3x-14}}{x-5}$

$$\lim_{x\to 5}\frac{\sqrt{5-4}-\sqrt{3\cdot 5-14}}{5-5}=\frac{\sqrt{1}-\sqrt{1}}{5-5}=\frac{0}{0}$$

$$\lim_{x\to 5}\frac{\sqrt{x-4}-\sqrt{3x-14}}{x-5}\cdot\frac{\sqrt{x-4}+\sqrt{3x-14}}{\sqrt{x-4}+\sqrt{3x-14}}$$

$$\lim_{x\to 5}\frac{\sqrt{x-4}^{2}-\sqrt{3x-14}^{2}}{(x-5)}\cdot\frac{1}{\sqrt{x-4}+\sqrt{3x-14}}$$

$$\lim_{x\to 5}\frac{x-4-3x+14}{(x-5)}\cdot\frac{1}{\sqrt{x-4}+\sqrt{3x-14}}$$

$$\lim_{x\to 5}\frac{-2x+10}{(x-5)}\cdot\frac{1}{\sqrt{x-4}+\sqrt{3x-14}}$$

$$\lim_{x\to 5}\frac{-2(x-5)}{(x-5)}\cdot\frac{1}{\sqrt{x-4}+\sqrt{3x-14}}$$

$$\lim_{x\to 5}\frac{-2}{1}\cdot\frac{1}{\sqrt{x-4}+\sqrt{3x-14}}=-2\cdot\frac{1}{\sqrt{5-4}+\sqrt{3\cdot 5-14}}$$

$$\lim_{x\to 5}-2\cdot\frac{1}{\sqrt{1}+\sqrt{1}}=-\frac{2}{2}$$

$$\lim_{x\to 5}\frac{\sqrt{x-4}-\sqrt{3x-14}}{x-5}=-1$$

28.-$\lim_{x\to 3}\frac{\sqrt{x^2-2x+6}-\sqrt{x^2+2x-6}}{x^2-4x+3}$

$$\lim_{x\to 3}\frac{\sqrt{3^2-2\cdot 3+6}-\sqrt{3^2+2\cdot 3-6}}{3^2-4\cdot 3+3}=\frac{0}{0}$$

$$\lim_{x\to 3}\frac{\sqrt{x^2-2x+6}-\sqrt{x^2+2x-6}}{x^2-4x+3}\cdot\frac{\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6}}{\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6}}$$

$$\lim_{x\to 3}\frac{\sqrt{x^2-2x+6}^2-\sqrt{x^2+2x-6}^2}{x^2-4x+3}\cdot\frac{1}{\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6}}$$

$$\lim_{x\to 3}\frac{x^2-2x+6-(x^2+2x-6)}{x^2-4x+3}\cdot\frac{1}{\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6}}$$

$$\lim_{x\to 3}\frac{x^2-2x+6-x^2-2x+6}{x^2-4x+3}\cdot\frac{1}{\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6}}$$

$$\lim_{x\to 3}\frac{-2x+6-2x+6}{x^2-4x+3}\cdot\frac{1}{\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6}}$$

$$\lim_{x\to 3}\frac{-4x+12}{x^2-4x+3}\cdot\frac{1}{\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6}}$$

$$\lim_{x\to 3}\frac{-4(x-3)}{x^2-4x+3}\cdot\frac{1}{\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6}}$$

$$\lim_{x\to 3}\frac{-4(x-3)}{(x-3)(x-1)}\cdot\frac{1}{\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6}}$$

$$\lim_{x\to 3}\frac{-4}{(x-1)}\cdot\frac{1}{\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6}}$$

$$\lim_{x\to 3}\frac{-4}{(3-1)}\cdot\frac{1}{\sqrt{3^2-2\cdot 3+6}+\sqrt{3^2+2\cdot 3-6}}$$

$$\lim_{x\to 3}\frac{-4}{(2)}\cdot\frac{1}{\sqrt{3^2-2\cdot 3+6}+\sqrt{3^2+2\cdot 3-6}}=\lim_{x\to 3}\frac{-4}{(2)}\cdot\frac{1}{\sqrt{3^2}+\sqrt{3^2}}$$

$$\lim_{x\to 3}\frac{-2}{6}=-\frac{1}{3}$$

$$\lim_{x\to 3}\frac{\sqrt{x^2-2x+6}-\sqrt{x^2+2x-6}}{x^2-4x+3}=-\frac{1}{3}$$

29.- $\lim_{x\to 2}\frac{3x-6}{1-\sqrt{4x-7}}$

$$\lim_{x\to 2}\frac{3\cdot 2-6}{1-\sqrt{4\cdot 2-7}}=\frac{0}{0}$$

$$\lim_{x\to 2}\frac{3x-6}{1-\sqrt{4x-7}}\cdot\frac{1+\sqrt{4x-7}}{1+\sqrt{4x-7}}$$

$$\lim_{x\to 2}\frac{3x-6}{1-\sqrt{4x-7}^2}\cdot\frac{1+\sqrt{4x-7}}{1}$$

$$\lim_{x\to 2}\frac{3x-6}{1-4x+7}\cdot\frac{1+\sqrt{4x-7}}{1}=\lim_{x\to 2}\frac{3x-6}{8-4x}\cdot\frac{1+\sqrt{4x-7}}{1}$$

$$\lim_{x\to 2}\frac{3(x-2)}{4(2-x)}\cdot\frac{1+\sqrt{4x-7}}{1}=\lim_{x\to 2}\frac{3(x-2)}{-4(x-2)}\cdot\frac{1+\sqrt{4x-7}}{1}$$

$$\lim_{x\to 2} -\frac{3}{4}\cdot\frac{1+\sqrt{4x-7}}{1}$$

$$\lim_{x\to 2} -\frac{3}{4}\cdot\frac{1+\sqrt{4\cdot 2-7}}{1} = \lim_{x\to 2} -\frac{3}{4}\cdot\frac{1+\sqrt{8-7}}{1}$$

$$\lim_{x\to 2} -\frac{3}{4}\cdot\frac{2}{1} = -\frac{3}{2}$$

$$\lim_{x\to 2}\frac{3x-6}{1-\sqrt{4x-7}} = -\frac{3}{2}$$

$30. - \lim_{x\to 0}\frac{\sqrt{x+a+b}-\sqrt{a+b}}{x}, a>0, b>0$

$$\lim_{x\to 0}\frac{\sqrt{o+a+b}-\sqrt{a+b}}{0} = \frac{0}{0}$$

$$\lim_{x\to 0}\frac{\sqrt{x+a+b}-\sqrt{a+b}}{x}\cdot\frac{\sqrt{x+a+b}+\sqrt{a+b}}{\sqrt{x+a+b}+\sqrt{a+b}}$$

$$\lim_{x\to 0}\frac{\sqrt{x+a+b}^2-\sqrt{a+b}^2}{x}\cdot\frac{1}{\sqrt{x+a+b}+\sqrt{a+b}}$$

$$\lim_{x\to 0}\frac{x+a+b-a-b}{x}\cdot\frac{1}{\sqrt{x+a+b}+\sqrt{a+b}}$$

$$\lim_{x\to 0}\frac{x}{x}\cdot\frac{1}{\sqrt{x+a+b}+\sqrt{a+b}}$$

$$\lim_{x\to 0}\frac{1}{\sqrt{x+a+b}+\sqrt{a+b}} = \frac{1}{\sqrt{0+a+b}+\sqrt{a+b}}$$

$$\lim_{x\to 0}\frac{1}{\sqrt{a+b}+\sqrt{a+b}}=\frac{1}{2\sqrt{a+b}}$$

$$\lim_{x\to 0}\frac{\sqrt{x+a+b}-\sqrt{a+b}}{x}=\frac{1}{2\sqrt{a+b}}$$

$$31.-\lim_{x\to 4}\frac{2-\sqrt{x}}{3-\sqrt{2x+1}}$$

$$\lim_{x\to 4}\frac{2-\sqrt{4}}{3-\sqrt{2\cdot 4+1}}=\frac{0}{0}$$

$$\lim_{x\to 4}\frac{2-\sqrt{x}}{3-\sqrt{2x+1}}\cdot\frac{2+\sqrt{x}}{2+\sqrt{x}}\cdot\frac{3+\sqrt{2x+1}}{3+\sqrt{2x+1}}$$

$$\lim_{x\to 4}\frac{2^2-\sqrt{x}^2}{3^2-\sqrt{2x+1}^2}\cdot\frac{1}{2+\sqrt{x}}\cdot\frac{3+\sqrt{2x+1}}{1}$$

$$\lim_{x\to 4}\frac{2^2-x}{3^2-2x+1}\cdot\frac{3+\sqrt{2x+1}}{2+\sqrt{x}}$$

$$\lim_{x\to 4}\frac{2^2-x}{8-2x}\cdot\frac{3+\sqrt{2x+1}}{2+\sqrt{x}}$$

$$\lim_{x\to 4}\frac{4-x}{2(4-x)}\cdot\frac{3+\sqrt{2x+1}}{2+\sqrt{x}}$$

$$\lim_{x\to 4}\frac{1}{2}\cdot\frac{3+\sqrt{2x+1}}{(2+\sqrt{x})}=\frac{1}{2}\cdot\frac{3+\sqrt{2\cdot 4+1}}{(2+\sqrt{4})}$$

$$\lim_{x\to 4}\frac{1}{2}\cdot\frac{3+\sqrt{9}}{2+\sqrt{4}}=\frac{1}{2}\cdot\frac{3+3}{(2+2)}=\frac{3}{4}$$

$$\lim_{x\to 4}\frac{2-\sqrt{x}}{3-\sqrt{2x+1}}=\frac{3}{4}$$

32.- $\lim_{x\to a}\frac{\sqrt{b^2-x}-\sqrt{b^2-a}}{x-a}$

$$\lim_{x\to a}\frac{\sqrt{b^2-a}-\sqrt{b^2-a}}{a-a}=\frac{0}{0}$$

$$\lim_{x\to a}\frac{\sqrt{b^2-x}-\sqrt{b^2-a}}{x-a}\cdot\frac{\sqrt{b^2-x}+\sqrt{b^2-a}}{\sqrt{b^2-x}+\sqrt{b^2-a}}$$

$$\lim_{x\to a}\frac{\sqrt{b^2-x}^{2}-\sqrt{b^2-a}^{2}}{x-a}\cdot\frac{1}{\sqrt{b^2-x}+\sqrt{b^2-a}}$$

$$\lim_{x\to a}\frac{b^2-x-b^2+a}{x-a}\cdot\frac{1}{\sqrt{b^2-x}+\sqrt{b^2-a}}$$

$$\lim_{x\to a}\frac{-x+a}{x-a}\cdot\frac{1}{\sqrt{b^2-x}+\sqrt{b^2-a}}$$

$$\lim_{x\to a}\frac{-(x-a)}{x-a}\cdot\frac{1}{\sqrt{b^2-x}+\sqrt{b^2-a}}$$

$$\lim_{x\to a}\frac{-1}{\sqrt{b^2-x}+\sqrt{b^2-a}}=\frac{-1}{\sqrt{b^2-a}+\sqrt{b^2-a}}$$

$$\lim_{x\to a} -\frac{1}{2\sqrt{b^2-a}}$$

$$\lim_{x\to a}\frac{\sqrt{b^2-x}-\sqrt{b^2-a}}{x-a} = -\frac{1}{2\sqrt{b^2-a}}$$

33.- $\lim_{x\to 0}\frac{\sqrt[3]{8+x}-2}{x}$

$$\lim_{x\to 0}\frac{\sqrt[3]{8+0}-2}{0} = \frac{0}{0}$$

$$\lim_{x\to 0}\frac{\sqrt[3]{8+x}-2}{x}\cdot\frac{\left(\sqrt[3]{8+x}\right)^2+2\cdot\sqrt[3]{8+x}+2^2}{\left(\sqrt[3]{8+x}\right)^2+2\cdot\sqrt[3]{8+x}+2^2}$$

$$\lim_{x\to 0}\frac{\sqrt[3]{8+x}^3-2^3}{x}\cdot\frac{1}{\left(\sqrt[3]{8+x}\right)^2+2\cdot\sqrt[3]{8+x}+2^2}$$

$$\lim_{x\to 0}\frac{8+x\ -8}{x}\cdot\frac{1}{\left(\sqrt[3]{8+x}\right)^2+2\cdot\sqrt[3]{8+x}+2^2}$$

$$\lim_{x\to 0}\frac{x}{x}\cdot\frac{1}{\left(\sqrt[3]{8+x}\right)^2+2\cdot\sqrt[3]{8+x}+2^2}$$

$$\lim_{x\to 0}\frac{1}{\left(\sqrt[3]{8+0}\right)^2+2\cdot\sqrt[3]{8+0}+2^2} = \frac{1}{\left(\sqrt[3]{8}\right)^2+2\cdot\sqrt[3]{8}+2^2}$$

$$\lim_{x\to 0}\frac{1}{2^2+2\cdot 2+2^2} = \frac{1}{4+4+4} = \frac{1}{12}$$

$$\lim_{x\to 0}\frac{\sqrt[3]{8+x}-2}{x}=\frac{1}{12}$$

34.- $\lim\limits_{x\to 0}\frac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1}$

$$\lim_{x\to 0}\frac{\sqrt{1+0}-1}{\sqrt[3]{1+0}-1}=\frac{0}{0}$$

$$\lim_{x\to 0}\frac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1}\cdot\frac{\sqrt{1+x}+1}{\sqrt{1+x}+1}\cdot\frac{\left(\sqrt[3]{1+x}\right)^2+\sqrt[3]{1+x}+1^2}{\left(\sqrt[3]{1+x}\right)^2+\sqrt[3]{1+x}+1^2}$$

$$\lim_{x\to 0}\frac{\sqrt{1+x}^2-1^2}{\sqrt[3]{1+x}^3-1^3}\cdot\frac{1}{\sqrt{1+x}+1}\cdot\frac{\left(\sqrt[3]{1+x}\right)^2+\sqrt[3]{1+x}+1^2}{1}$$

$$\lim_{x\to 0}\frac{1+x-1^2}{1+x-1^3}\cdot\frac{1}{\sqrt{1+x}+1}\cdot\frac{\left(\sqrt[3]{1+x}\right)^2+\sqrt[3]{1+x}+1^2}{1}$$

$$\lim_{x\to 0}\frac{x}{x}\cdot\frac{1}{\sqrt{1+x}+1}\cdot\frac{\left(\sqrt[3]{1+x}\right)^2+\sqrt[3]{1+x}+1^2}{1}$$

$$\lim_{x\to 0}\frac{1}{\sqrt{1+x}+1}\cdot\frac{\left(\sqrt[3]{1+x}\right)^2+\sqrt[3]{1+x}+1^2}{1}$$

$$\lim_{x\to 0}\frac{\left(\sqrt[3]{1+x}\right)^2+\sqrt[3]{1+x}+1^2}{\sqrt{1+x}+1}=\frac{\left(\sqrt[3]{1+0}\right)^2+\sqrt[3]{1+0}+1^2}{\sqrt{1+0}+1}$$

$$\lim_{x\to 0}\frac{1^2+\sqrt[3]{1}+1^2}{\sqrt{1}+1}=\frac{3}{2}$$

$$\lim_{x\to 0}\frac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1}=\frac{3}{2}$$

35.-$\lim_{x\to 8}\frac{\sqrt[3]{x}-2}{x-8}$

$$\lim_{x\to 8}\frac{\sqrt[3]{8}-2}{8-8}=\frac{0}{0}$$

$$\lim_{x\to 8}\frac{\sqrt[3]{x}-2}{x-8}\cdot\frac{\sqrt[3]{x}^2+2\sqrt[3]{x}+2^2}{\sqrt[3]{x}^2+2\sqrt[3]{x}+2^2}$$

$$\lim_{x\to 8}\frac{\sqrt[3]{x}^3-2^3}{x-8}\cdot\frac{1}{\sqrt[3]{x}^2+2\sqrt[3]{x}+2^2}$$

$$\lim_{x\to 8}\frac{x-8}{x-8}\cdot\frac{1}{\sqrt[3]{x}^2+2\sqrt[3]{x}+2^2}=\frac{1}{\sqrt[3]{8}^2+2\sqrt[3]{8}+2^2}$$

$$\lim_{x\to 8}\frac{1}{\sqrt[3]{8}^2+2\sqrt[3]{8}+2^2}=\frac{1}{2^2+2\cdot 2+2^2}$$

$$\lim_{x\to 8}\frac{1}{2^2+2\cdot 2+2^2}=\frac{1}{12}$$

$$\lim_{x\to 8}\frac{\sqrt[3]{x}-2}{x-8}=\frac{1}{12}$$

36.- $\lim_{x\to 0}\frac{\sqrt[3]{(x+1)^2}-2\sqrt[3]{x+1}+1}{x^2}$

$$\lim_{x\to 0}\frac{\sqrt[3]{(0+1)^2}-2\sqrt[3]{0+1}+1}{0^2}=\frac{0}{0}$$

$$\lim_{x\to 0}\frac{\sqrt[3]{(x+1)^2}-2\sqrt[3]{x+1}+1}{x^2}$$

$$\lim_{x\to 0}\frac{\left(\sqrt[3]{x+1}-1\right)^2}{x^2}\cdot\frac{\left(\sqrt[3]{(x+1)^2}+\sqrt[3]{x+1}+1\right)^2}{\left(\sqrt[3]{(x+1)^2}+\sqrt[3]{x+1}+1\right)^2}$$

$$\lim_{x\to 0}\frac{\left(\sqrt[3]{x+1}^3-1^3\right)^2}{x^2}\cdot\frac{1}{\left(\sqrt[3]{(x+1)^2}+\sqrt[3]{x+1}+1\right)^2}$$

$$\lim_{x\to 0}\frac{(x+1-1)^2}{x^2}\cdot\frac{1}{\left(\sqrt[3]{(x+1)^2}+\sqrt[3]{x+1}+1\right)^2}$$

$$\lim_{x\to 0}\frac{x^2}{x^2}\cdot\frac{1}{\left(\sqrt[3]{(x+1)^2}+\sqrt[3]{x+1}+1\right)^2}=\frac{1}{\left(\sqrt[3]{(x+1)^2}+\sqrt[3]{x+1}+1\right)^2}$$

$$\lim_{x\to 0}\frac{1}{\left(\sqrt[3]{(0+1)^2}+\sqrt[3]{0+1}+1\right)^2}=\frac{1}{(1+1+1)^2}=\frac{1}{9}$$

$$\lim_{x\to 0}\frac{\sqrt[3]{(x+1)^2}-2\sqrt[3]{x+1}+1}{x^2}=\frac{1}{9}$$

37.- $\lim_{x\to1}\frac{\sqrt{x}+\sqrt{4x+5}-\sqrt{3x+13}}{x-1}$

$$\lim_{x\to1}\frac{\sqrt{1}+\sqrt{4\cdot1+5}-\sqrt{3\cdot1+13}}{1-1}=\frac{0}{0}$$

$$\lim_{x\to1}\frac{(\sqrt{x}-1)+(\sqrt{4x+5}-3)-(\sqrt{3x+13}-4)}{x-1}$$

$$\lim_{x\to1}\frac{\sqrt{x}-1}{x-1}+\frac{\sqrt{4x+5}-3}{x-1}-\frac{\sqrt{3x+13}-4}{x-1}$$

$$\lim_{x\to1}\frac{\sqrt{x}-1}{x-1}\cdot\frac{\sqrt{x}+1}{\sqrt{x}+1}+\frac{\sqrt{4x+5}-3}{x-1}\cdot\frac{\sqrt{4x+5}+3}{\sqrt{4x+5}+3}-\frac{\sqrt{3x+13}-4}{x-1}\cdot\frac{\sqrt{3x+13}+4}{\sqrt{3x+13}+4}$$

$$\lim_{x\to1}\frac{\sqrt{x}^2-1^2}{x-1}\cdot\frac{1}{\sqrt{x}+1}+\frac{\sqrt{4x+5}^2-3^2}{x-1}\cdot\frac{1}{\sqrt{4x+5}+3}-\frac{\sqrt{3x+13}^2-4^2}{x-1}\cdot\frac{1}{\sqrt{3x+13}+4}$$

$$\lim_{x\to1}\frac{x-1}{x-1}\cdot\frac{1}{\sqrt{x}+1}+\frac{4x+5-9}{x-1}\cdot\frac{1}{\sqrt{4x+5}+3}-\frac{3x+13-16}{x-1}\cdot\frac{1}{\sqrt{3x+13}+4}$$

$$\lim_{x\to1}\frac{1}{\sqrt{x}+1}+\frac{4x-4}{x-1}\cdot\frac{1}{\sqrt{4x+5}+3}-\frac{3x-3}{x-1}\cdot\frac{1}{\sqrt{3x+13}+4}$$

$$\lim_{x\to 1}\frac{1}{\sqrt{x}+1}+\frac{4(x-1)}{x-1}\cdot\frac{1}{\sqrt{4x+5}+3}-\frac{3(x-1)}{x-1}\cdot\frac{1}{\sqrt{3x+13}+4}$$

$$\lim_{x\to 1}\frac{1}{\sqrt{x}+1}+\frac{4}{\sqrt{4x+5}+3}-\frac{3}{\sqrt{3x+13}+4}$$

$$\lim_{x\to 1}\frac{1}{\sqrt{1}+1}+\frac{4}{\sqrt{4\cdot 1+5}+3}-\frac{3}{\sqrt{3\cdot 1+13}+4}=\frac{1}{2}+\frac{4}{3+3}-\frac{3}{4+4}$$

$$\lim_{x\to 1}\frac{1}{2}+\frac{4}{6}-\frac{3}{8}=\frac{1}{2}+\frac{2}{3}-\frac{3}{8}=\frac{12+16-9}{24}=\frac{19}{24}$$

$$\lim_{x\to 1}\frac{\sqrt{x}+\sqrt{4x+5}-\sqrt{3x+13}}{x-1}=\frac{19}{24}$$

38.- $\lim_{x\to 2a}\frac{\sqrt{x}-\sqrt{2a}+\sqrt{x-2a}}{\sqrt{x^2-4a^2}}$

$$\lim_{x\to 2a}\frac{\sqrt{2a}-\sqrt{2a}+\sqrt{2a-2a}}{\sqrt{(2a)^2-4a^2}}=\frac{0}{0}$$

$$\lim_{x\to 2a}\frac{(\sqrt{x}-\sqrt{2a})}{\sqrt{x^2-4a^2}}+\frac{\sqrt{x-2a}}{\sqrt{x^2-4a^2}}$$

$$\lim_{x\to 2a}\frac{\sqrt{x}-\sqrt{2a}}{\sqrt{x^2-4a^2}}\cdot\frac{\sqrt{x^2-4a^2}}{\sqrt{x^2-4a^2}}+\sqrt{\frac{(x-2a)}{(x-2a)(x+2a)}}$$

$$\lim_{x \to 2a} \frac{\sqrt{x} - \sqrt{2a}}{\sqrt{x^2 - 4a^2}^2} \cdot \frac{\sqrt{x^2 - 4a^2}}{1} + \sqrt{\frac{1}{x + 2a}}$$

$$\lim_{x \to 2a} \frac{\sqrt{x} - \sqrt{2a}}{x^2 - 4a^2} \cdot \frac{\sqrt{x^2 - 4a^2}}{1} + \frac{1}{\sqrt{x + 2a}}$$

$$\lim_{x \to 2a} \frac{\sqrt{x} - \sqrt{2a}}{x^2 - 4a^2} \cdot \frac{\sqrt{x^2 - 4a^2}}{1} \cdot \frac{\sqrt{x} + \sqrt{2a}}{\sqrt{x} + \sqrt{2a}} + \frac{1}{\sqrt{x + 2a}}$$

$$\lim_{x \to 2a} \frac{\sqrt{x}^2 - \sqrt{2a}^2}{x^2 - 4a^2} \cdot \frac{\sqrt{x^2 - 4a^2}}{1} \cdot \frac{1}{\sqrt{x} + \sqrt{2a}} + \frac{1}{\sqrt{x + 2a}}$$

$$\lim_{x \to 2a} \frac{x - 2a}{x^2 - 4a^2} \cdot \frac{\sqrt{x^2 - 4a^2}}{1} \cdot \frac{1}{\sqrt{x} + \sqrt{2a}} + \frac{1}{\sqrt{x + 2a}}$$

$$\lim_{x \to 2a} \frac{x - 2a}{x^2 - 4a^2} \cdot \frac{\sqrt{x^2 - 4a^2}}{\sqrt{x} + \sqrt{2a}} + \frac{1}{\sqrt{x + 2a}}$$

$$\lim_{x \to 2a} \frac{(x - 2a)}{(x - 2a)(x + 2a)} \cdot \frac{\sqrt{x^2 - 4a^2}}{\sqrt{x} + \sqrt{2a}} + \frac{1}{\sqrt{x + 2a}}$$

$$\lim_{x \to 2a} \frac{1}{(x + 2a)} \cdot \frac{\sqrt{x^2 - 4a^2}}{\sqrt{x} + \sqrt{2a}} + \frac{1}{\sqrt{x + 2a}}$$

$$\lim_{x\to 2a} \frac{1}{(2a+2a)} \cdot \frac{\sqrt{(2a)^2-4a^2}}{\sqrt{2a}+\sqrt{2a}} + \frac{1}{\sqrt{2a+2a}}$$

$$\lim_{x\to 2a} \frac{1}{(2a+2a)} \cdot \frac{\sqrt{(2a)^2-4a^2}}{\sqrt{2a}+\sqrt{2a}} + \frac{1}{\sqrt{2a+2a}}$$

$$\lim_{x\to 2a} \frac{1}{4a} \cdot \frac{0}{\sqrt{2a}+\sqrt{2a}} + \frac{1}{\sqrt{2a+2a}}$$

$$\lim_{x\to 2a} \frac{1}{\sqrt{2a+2a}} = \lim_{x\to 2a} \frac{1}{\sqrt{4a}} = \frac{1}{2\sqrt{a}}$$

$$\lim_{x\to 2a} \frac{\sqrt{x}-\sqrt{2a}+\sqrt{x-2a}}{\sqrt{x^2-4a^2}} = \frac{1}{2\sqrt{a}}$$

39. $\lim_{x\to 1} \frac{\sqrt{x^2+x+7}-\sqrt{2x^2+10x-3}}{\sqrt{x^2+1}-\sqrt{3x^2-1}}$

$$\lim_{x\to 1} \frac{\sqrt{1^2+1+7}-\sqrt{2\cdot 1^2+10\cdot 1-3}}{\sqrt{1^2+1}-\sqrt{3\cdot 1^2-1}} = \frac{\sqrt{9}-\sqrt{9}}{\sqrt{2}-\sqrt{2}} = \frac{0}{0}$$

$$\lim_{x\to 1} \frac{\sqrt{x^2+x+7}-\sqrt{2x^2+10x-3}}{\sqrt{x^2+1}-\sqrt{3x^2-1}} \cdot \frac{\sqrt{x^2+x+7}+\sqrt{2x^2+10x-3}}{\sqrt{x^2+x+7}+\sqrt{2x^2+10x-3}} \cdot \frac{\sqrt{x^2+1}+\sqrt{3x^2-1}}{\sqrt{x^2+1}+\sqrt{3x^2-1}}$$

$$\lim_{x\to 1}\frac{\sqrt{x^2+x+7}^2-\sqrt{2x^2+10x-3}^2}{\sqrt{x^2+1}^2-\sqrt{3x^2-1}^2}\cdot\frac{1}{\sqrt{x^2+x+7}+\sqrt{2x^2+10x-3}}\cdot\frac{\sqrt{x^2+1}+\sqrt{3x^2-1}}{1}$$

$$\lim_{x\to 1}\frac{\sqrt{x^2+x+7}^2-\sqrt{2x^2+10x-3}^2}{\sqrt{x^2+1}^2-\sqrt{3x^2-1}^2}\cdot\frac{1}{\sqrt{x^2+x+7}+\sqrt{2x^2+10x-3}}\cdot\frac{\sqrt{x^2+1}+\sqrt{3x^2-1}}{1}$$

$$\lim_{x\to 1}\frac{x^2+x+7-2x^2-10x+3}{x^2+1-3x^2+1}\cdot\frac{1}{\sqrt{x^2+x+7}+\sqrt{2x^2+10x-3}}\cdot\frac{\sqrt{x^2+1}+\sqrt{3x^2-1}}{1}$$

$$\lim_{x\to 1}\frac{-x^2-9x+10}{-2x^2+2}\cdot\frac{1}{\sqrt{x^2+x+7}+\sqrt{2x^2+10x-3}}\cdot\frac{\sqrt{x^2+1}+\sqrt{3x^2-1}}{1}$$

$$\lim_{x\to 1}\frac{-(x^2+9x-10)}{-2(x^2-1)}\cdot\frac{1}{\sqrt{x^2+x+7}+\sqrt{2x^2+10x-3}}\cdot\frac{\sqrt{x^2+1}+\sqrt{3x^2-1}}{1}$$

$$\lim_{x\to 1}\frac{(x+10)(x-1)}{(x-1)(x+1)}\cdot\frac{1}{\sqrt{x^2+x+7}+\sqrt{2x^2+10x-3}}\cdot\frac{\sqrt{x^2+1}+\sqrt{3x^2-1}}{1}$$

$$\lim_{x\to 1}\frac{(x+10)}{(x+1)}\cdot\frac{1}{\sqrt{x^2+x+7}+\sqrt{2x^2+10x-3}}\cdot\frac{\sqrt{x^2+1}+\sqrt{3x^2-1}}{1}$$

$$\lim_{x\to 1}\frac{(1+10)}{(1+1)}\cdot\frac{1}{\sqrt{1^2+1+7}+\sqrt{2\cdot 1^2+10\cdot 1-3}}\cdot\frac{\sqrt{1^2+1}+\sqrt{3\cdot 1^2-1}}{1}$$

$$\lim_{x\to 1}\frac{11}{2}\cdot\frac{1}{\sqrt{9}+\sqrt{9}}\cdot\frac{\sqrt{2}+\sqrt{2}}{1}=\frac{11}{2}\cdot\frac{2\sqrt{2}}{2\sqrt{9}}=\frac{11}{2}\cdot\sqrt{2}$$

$$\lim_{x\to 1}\frac{\sqrt{x^2+x+7}-\sqrt{2x^2+10x-3}}{\sqrt{x^2+1}-\sqrt{3x^2-1}}=\frac{11}{2}\cdot\sqrt{2}$$

40.- $\lim_{x\to a}\frac{\sqrt[m]{x}-\sqrt[m]{a}}{x-a}$

$$\lim_{x\to a}\frac{\sqrt[m]{a}-\sqrt[m]{a}}{a-a}=\frac{0}{0}$$

$$\lim_{x\to a}\frac{\sqrt[m]{x}-\sqrt[m]{a}}{x-a}=\lim_{x\to a}\frac{\sqrt[m]{x}-\sqrt[m]{a}}{\sqrt[m]{x}^{m}-\sqrt[m]{a}^{m}}$$

$$\lim_{x\to a}\frac{\sqrt[m]{x}-\sqrt[m]{a}}{(\sqrt[m]{x}-\sqrt[m]{a})\left[(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}a+(\sqrt[m]{x})^{m-3}a^2+\cdots+(\sqrt[m]{a})^{m-1}\right]}$$

$$\lim_{x\to a}\frac{1}{\left[\left(\sqrt[m]{x}\right)^{m-1}+\left(\sqrt[m]{x}\right)^{m-2}a+\left(\sqrt[m]{x}\right)^{m-3}a^{2}+\cdots+\left(\sqrt[m]{a}\right)^{m-1}\right]}$$

$$\lim_{x\to a}\frac{1}{\left[\left(\sqrt[m]{a}\right)^{m-1}+\left(\sqrt[m]{a}\right)^{m-1}+\left(\sqrt[m]{a}\right)^{m-1}+\cdots+\left(\sqrt[m]{a}\right)^{m-1}\right]}$$

$$\lim_{x\to a}\ \frac{1}{m\left(\sqrt[m]{a}\ \right)^{m-1}}=\frac{1}{ma^{\frac{m-1}{m}}}=\frac{a^{-\frac{m-1}{m}}}{m}$$

$$\lim_{x\to a}\ \frac{a^{\frac{1-m}{m}}}{m}=\frac{\sqrt[m]{a^{1-m}}}{m}$$

$$\lim_{x\to a}\frac{\sqrt[m]{x}-\sqrt[m]{a}}{x-a}=\frac{\sqrt[m]{a^{1-m}}}{m}$$

1.4 Calculo de Limites con Radicales con la Calculadora

$$41.-\lim_{x\to 0}\frac{\sqrt{1+x^{2}}-1}{x^{2}}$$

$$\lim_{x\to 0}\frac{\sqrt{1+0^{2}}-1}{0^{2}}=\frac{0}{0}$$

$$\lim_{x \to 0} \frac{\sqrt{1+x^2}-1}{x^2} = \frac{1}{2}$$

42.-$\lim_{x \to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{x}$

$$\lim_{x \to 0} \frac{\sqrt{1+0}-\sqrt{1+0}}{0} = \frac{0}{0}$$

$$\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}=1$$

43 $\lim_{x\to 5}\frac{\sqrt{x-4}-\sqrt{3x-14}}{x-5}$

$$\lim_{x\to 5}\frac{\sqrt{5-4}-\sqrt{3\cdot 5-14}}{5-5}=\frac{\sqrt{1}-\sqrt{1}}{5-5}=\frac{0}{0}$$

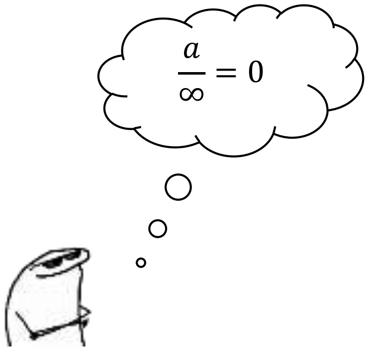
$$\lim_{x\to 5}\frac{\sqrt{x-4}-\sqrt{3x-14}}{x-5}=-1$$

1.5 Límites al Infinito

44.- $\lim_{x \to \infty} \frac{x^3+2x^2+3x+4}{4x^3+3x^2+2x+1}$

$$\lim_{x \to \infty} \frac{\infty^3 + 2 \cdot \infty^2 + 3 \cdot \infty + 4}{4 \cdot \infty^3 + 3 \cdot \infty^2 + 2 \cdot \infty + 1} = \frac{\infty}{\infty}$$

$$\lim_{x \to \infty} \frac{\frac{x^3 + 2x^2 + 3x + 4}{x^3}}{\frac{4x^3 + 3x^2 + 2x + 1}{x^3}}$$



$$\lim_{x \to \infty} \frac{\frac{x^3}{x^3} + \frac{2x^2}{x^3} + \frac{3x}{x^3} + \frac{4}{x^3}}{\frac{4x^3}{x^3} + \frac{3x^2}{x^3} + \frac{2x}{x^3} + \frac{1}{x^3}}$$

$$\lim_{x \to \infty} \frac{1 + \frac{2}{x} + \frac{3}{x^2} + \frac{4}{x^3}}{4 + \frac{3}{x} + \frac{2}{x^2} + \frac{1}{x^3}}$$

$$\lim_{x \to \infty} \frac{1 + \frac{2}{\infty} + \frac{3}{\infty^2} + \frac{4}{\infty^3}}{4 + \frac{3}{\infty} + \frac{2}{\infty^2} + \frac{1}{\infty^3}} = \lim_{x \to \infty} \frac{1 + 0 + 0 + 0}{4 + 0 + 0 + 0} = \frac{1}{4}$$

$$\lim_{x \to \infty} \frac{x^3 + 2x^2 + 3x + 4}{4x^3 + 3x^2 + 2x + 1} = \frac{1}{4}$$

45.- $\lim_{x \to \infty} \frac{4x^3+7x+5}{-8x^3+x+2}$

$$\lim_{x \to \infty} \frac{4 \cdot \infty^3 + 7 \cdot \infty + 5}{-8 \cdot \infty^3 + \infty + 2} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty}\frac{\frac{4x^3+7x+5}{x^3}}{\frac{-8x^3+x+2}{x^3}}$$

$$\lim_{x\to\infty}\frac{\frac{4x^3}{x^3}+\frac{7x}{x^3}+\frac{5}{x^3}}{\frac{-8x^3}{x^3}+\frac{x}{x^3}+\frac{2}{x^3}}=\lim_{x\to\infty}\frac{4+\frac{7}{x^2}+\frac{5}{x^3}}{-8+\frac{1}{x^2}+\frac{2}{x^3}}$$

$$\lim_{x\to\infty}\frac{4+\frac{7}{\infty}+\frac{5}{\infty}}{-8+\frac{1}{\infty}+\frac{2}{\infty}}=\frac{4+0+0}{-8+0+0}=-\frac{4}{8}=-\frac{1}{2}$$

$$\lim_{x\to\infty}\frac{4x^3+7x+5}{-8x^3+x+2}=-\frac{1}{2}$$

46.- $\lim_{x\to\infty}\left(\frac{x^3}{x^2+2}-\frac{x^2}{x+2}\right)$

$$\lim_{x\to\infty}\left(\frac{\infty^3}{\infty^2+2}-\frac{\infty^2}{\infty+2}\right)=\infty-\infty$$

$$\lim_{x\to\infty}\left(\frac{x^3(x+2)-x^2(x^2+2)}{(x^2+2)(x+2)}\right)$$

$$\lim_{x\to\infty}\left(\frac{x^4+2x^3-x^4-2x^2}{(x^2+2)(x+2)}\right)$$

$$\lim_{x\to\infty}\left(\frac{2x^3-2x^2}{x^3+2x^2+2x+4}\right)=\frac{\frac{2x^3-2x^2}{x^3}}{\frac{x^3+2x^2+2x+4}{x^3}}$$

$$\lim_{x\to\infty}\frac{\frac{2x^3}{x^3}-\frac{2x^2}{x^3}}{\frac{x^3}{x^3}+\frac{2x^2}{x^3}+\frac{2x}{x^3}+\frac{4}{x^3}}=\frac{2-\frac{2}{x}}{1+\frac{2}{x}+\frac{2}{x^2}+\frac{4}{x^3}}$$

$$\lim_{x\to\infty}\frac{2-\frac{2}{\infty}}{1+\frac{2}{\infty}+\frac{2}{\infty^2}+\frac{4}{\infty^3}}=\frac{2-0}{1+0+0+0}=2$$

$$\lim_{x\to\infty}\left(\frac{x^3}{x^2+2}-\frac{x^2}{x+2}\right)=2$$

47.- $\lim\limits_{x\to\infty}\left(\frac{3x^2-2}{2x+1}\div\frac{x^2-4x}{x-3}\right)$

$$\lim_{x\to\infty}\left(\frac{(3x^2-2)}{(2x+1)}\cdot\frac{(x-3)}{(x^2-4x)}\right)$$

$$\lim_{x\to\infty}\left(\frac{3x^3-9x^2-2x+6}{2x^3-7x^2-4x}\right)$$

$$\lim_{x\to\infty}\frac{\frac{3x^3-9x^2-2x+6}{x^3}}{\frac{2x^3-7x^2-4x}{x^3}}=\frac{\frac{3x^3}{x^3}-\frac{9x^2}{x^3}-\frac{2x}{x^3}+\frac{6}{x^3}}{\frac{2x^3}{x^3}-\frac{7x^2}{x^3}-\frac{4x}{x^3}}$$

$$\lim_{x\to\infty}\frac{3-\frac{9}{x}-\frac{2}{x^2}+\frac{6}{x^3}}{2-\frac{7}{x}-\frac{4}{x^2}}=\frac{3-0-0+0}{2-0-0}=\frac{3}{2}$$

$$\lim_{x\to\infty}\left(\frac{3x^2-2}{2x+1}\div\frac{x^2-4x}{x-3}\right)=\frac{3}{2}$$

48.- $\lim_{x\to\infty}\sqrt{16x^2+8x+6}-\sqrt{16x^2-8x-6}$

$$\lim_{x\to\infty}\sqrt{16\cdot\infty^2+8\cdot\infty+6}-\sqrt{16\cdot\infty^2-8\cdot\infty-6}=\infty-\infty$$

$$\lim_{x\to\infty}\sqrt{16x^2+8x+6}-\sqrt{16x^2-8x-6}\cdot\frac{\sqrt{16x^2+8x+6}+\sqrt{16x^2-8x-6}}{\sqrt{16x^2+8x+6}+\sqrt{16x^2-8x-6}}$$

$$\lim_{x\to\infty}\frac{\sqrt{16x^2+8x+6}^2-\sqrt{16x^2-8x-6}^2}{\sqrt{16x^2+8x+6}+\sqrt{16x^2-8x-6}}$$

$$\lim_{x\to\infty}\frac{16x^2+8x+6-16x^2+8x+6}{\sqrt{16x^2+8x+6}+\sqrt{16x^2-8x-6}}$$

$$\lim_{x\to\infty}\frac{16x+12}{\sqrt{16x^2+8x+6}+\sqrt{16x^2-8x-6}}$$

$$\lim_{x\to\infty}\frac{4(4x+3)}{\sqrt{16x^2+8x+6}+\sqrt{16x^2-8x-6}}$$

$$4\lim_{x\to\infty}\frac{\frac{4x+3}{x}}{\frac{\sqrt{16x^2+8x+6}+\sqrt{16x^2-8x-6}}{\sqrt{x^2}}}$$

$$4\lim_{x\to\infty}\frac{\frac{4x+3}{x}}{\frac{\sqrt{16x^2+8x+6}}{\sqrt{x^2}}+\frac{\sqrt{16x^2-8x-6}}{\sqrt{x^2}}}$$

$$4\lim_{x\to\infty}\frac{\frac{4x}{x}+\frac{3}{x}}{\sqrt{\frac{16x^2+8x+6}{x^2}}+\sqrt{\frac{16x^2-8x-6}{x^2}}}$$

$$4\lim_{x\to\infty}\frac{4+\frac{3}{x}}{\sqrt{\frac{16x^2}{x^2}+\frac{8x}{x^2}+\frac{6}{x^2}}+\sqrt{\frac{16x^2}{x^2}-\frac{8x}{x^2}-\frac{6}{x^2}}}$$

$$4\lim_{x\to\infty}\frac{4+\frac{3}{x}}{\sqrt{16+\frac{8}{x}+\frac{6}{x^2}}+\sqrt{16-\frac{8}{x}-\frac{6}{x^2}}}=\frac{4+\frac{3}{\infty}}{\sqrt{16+\frac{8}{\infty}+\frac{6}{\infty}}+\sqrt{16-\frac{8}{\infty}-\frac{6}{\infty}}}$$

$$4\lim_{x\to\infty}\frac{4}{\sqrt{16}+\sqrt{16}}=\frac{4}{4+4}=4\cdot\frac{4}{8}=2$$

$$\lim_{x\to\infty}\sqrt{16x^2+8x+6}-\sqrt{16x^2-8x-6}=2$$

49.- $\lim_{x\to\infty} \frac{\sqrt{2x^2+1}}{x+3}$

$$\lim_{x\to\infty} \frac{\sqrt{2\cdot\infty^2+1}}{\infty+3} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{\sqrt{2x^2+1}}{x+3} = \lim_{x\to\infty} \frac{\frac{\sqrt{2x^2+1}}{\sqrt{x^2}}}{\frac{x+3}{x}}$$

$$\lim_{x\to\infty} \frac{\sqrt{\frac{2x^2+1}{x^2}}}{\frac{x}{x}+\frac{3}{x}} = \frac{\sqrt{\frac{2x^2}{x^2}+\frac{1}{x^2}}}{\frac{x}{x}+\frac{3}{x}} = \frac{\sqrt{2+\frac{1}{x^2}}}{1+\frac{3}{x}}$$

$$\lim_{x\to\infty} \frac{\sqrt{2+\frac{1}{\infty}}}{1+\frac{3}{\infty}} = \frac{\sqrt{2+0}}{1+0} = \sqrt{2}$$

$$\lim_{x\to\infty} \frac{\sqrt{2x^2+1}}{x+3} = \sqrt{2}$$

50.- $\lim_{x\to\infty} \sqrt{(x+a)(x+b)} - x$

$$\lim_{x\to\infty} \sqrt{(\infty+a)(\infty+b)} - \infty = \infty - \infty$$

$$\lim_{x\to\infty} \sqrt{(x+a)(x+b)} - x \cdot \frac{\sqrt{(x+a)(x+b)}+x}{\sqrt{(x+a)(x+b)}+x}$$

$$\lim_{x\to\infty} \frac{\sqrt{(x+a)(x+b)}^2 - x^2}{\sqrt{(x+a)(x+b)}+x} = \lim_{x\to\infty} \frac{(x+a)(x+b) - x^2}{\sqrt{(x+a)(x+b)}+x}$$

$$\lim_{x\to\infty} \frac{x^2 + xb + ax + ab - x^2}{\sqrt{(x+a)(x+b)}+x} = \lim_{x\to\infty} \frac{xb + ax + ab}{\sqrt{(x+a)(x+b)}+x}$$

$$\lim_{x\to\infty} \frac{\frac{xb + ax + ab}{x}}{\frac{\sqrt{(x+a)(x+b)}+x}{\sqrt{x^2}}} = \lim_{x\to\infty} \frac{\frac{xb}{x} + \frac{ax}{x} + \frac{ab}{x}}{\sqrt{\frac{(x+a)(x+b)}{x^2}} + \frac{x}{x}}$$

$$\lim_{x\to\infty} \frac{b + a + \frac{ab}{x}}{\sqrt{\frac{(x+a)(x+b)}{x^2}} + 1} = \lim_{x\to\infty} \frac{b + a + \frac{ab}{x}}{\sqrt{\frac{(x+a)}{x} \cdot \frac{(x+b)}{x}} + 1}$$

$$\lim_{x\to\infty} \frac{b + a + \frac{ab}{x}}{\sqrt{\left(\frac{x}{x} + \frac{a}{x}\right)\left(\frac{x}{x} + \frac{b}{x}\right)} + 1} = \lim_{x\to\infty} \frac{b + a + \frac{ab}{x}}{\sqrt{\left(1 + \frac{a}{x}\right)\left(1 + \frac{b}{x}\right)} + 1}$$

$$\lim_{x\to\infty} \frac{b + a + \frac{ab}{\infty}}{\sqrt{\left(1 + \frac{a}{\infty}\right)\left(1 + \frac{b}{\infty}\right)} + 1} = \frac{b+a}{\sqrt{1}+1}$$

$$\lim_{x\to\infty} \frac{a+b}{2}$$

$$\lim_{x\to\infty} \sqrt{(x+a)(x+b)} - x = \frac{a+b}{2}$$

51.- $\lim_{x\to\infty}\sqrt{x+\sqrt{2x}}-\sqrt{x-\sqrt{2x}}$

$$\lim_{x\to\infty}\sqrt{\infty+\sqrt{2\cdot\infty}}-\sqrt{\infty-\sqrt{2\cdot\infty}}$$

$$\lim_{x\to\infty}\sqrt{x+\sqrt{2x}}-\sqrt{x-\sqrt{2x}}\cdot\frac{\sqrt{x+\sqrt{2x}}+\sqrt{x-\sqrt{2x}}}{\sqrt{x+\sqrt{2x}}+\sqrt{x-\sqrt{2x}}}$$

$$\lim_{x\to\infty}\frac{\sqrt{x+\sqrt{2x}}^2-\sqrt{x-\sqrt{2x}}^2}{\sqrt{x+\sqrt{2x}}+\sqrt{x-\sqrt{2x}}}=\lim_{x\to\infty}\frac{2\sqrt{2x}}{\sqrt{x+\sqrt{2x}}+\sqrt{x-\sqrt{2x}}}$$

$$\lim_{x\to\infty}\frac{\frac{2\sqrt{2x}}{\sqrt{x}}}{\frac{\sqrt{x+\sqrt{2x}}+\sqrt{x-\sqrt{2x}}}{\sqrt{x}}}=\lim_{x\to\infty}\frac{\frac{2\sqrt{2x}}{\sqrt[4]{x}}}{\frac{\sqrt{x+\sqrt{2x}}}{\sqrt{x}}+\frac{\sqrt{x-\sqrt{2x}}}{\sqrt{x}}}$$

$$\lim_{x\to\infty}\frac{2\cdot\sqrt{\frac{2x}{x}}}{\sqrt{\left(\frac{x}{x}+\frac{\sqrt{2x}}{x}\right)}+\sqrt{\left(\frac{x}{x}-\frac{\sqrt{2x}}{x}\right)}}=\lim_{x\to\infty}\frac{2\cdot\sqrt{2}}{\sqrt{\left(1+\frac{\sqrt{2}}{x\cdot x^{-\frac{1}{2}}}\right)}+\sqrt{\left(1-\frac{\sqrt{2}}{x\cdot x^{-\frac{1}{2}}}\right)}}$$

$$\lim_{x\to\infty}\frac{2\cdot\sqrt{2}}{\sqrt{\left(1+\frac{\sqrt{2}}{\sqrt{x}}\right)}+\sqrt{\left(1-\frac{\sqrt{2}}{\sqrt{x}}\right)}}=\lim_{x\to\infty}\frac{2\cdot\sqrt{2}}{\sqrt{\left(1+\frac{\sqrt{2}}{\infty}\right)}+\sqrt{\left(1-\frac{\sqrt{2}}{\infty}\right)}}$$

$$\lim_{x\to\infty}\frac{2\cdot\sqrt{2}}{\sqrt{(1+1)}+\sqrt{(1-1)}}=\frac{2\cdot\sqrt{2}}{\sqrt{1}+\sqrt{1}}=\frac{2\cdot\sqrt{2}}{2}=\sqrt{2}$$

$$\lim_{x\to\infty}\sqrt{x+\sqrt{2x}}-\sqrt{x-\sqrt{2x}}=\sqrt{2}$$

52.- $\lim\limits_{x\to\infty}\frac{\sqrt{x}+\sqrt[3]{x}+\sqrt[4]{x}}{\sqrt{2x+1}}$

$$\lim_{x\to\infty}\frac{\sqrt{\infty}+\sqrt[3]{\infty}+\sqrt[4]{\infty}}{\sqrt{2\cdot\infty}+1}=\frac{\infty}{\infty}$$

$$\lim_{x\to\infty}\frac{\frac{\sqrt{x}+\sqrt[3]{x}+\sqrt[4]{x}}{\sqrt{x}}}{\frac{\sqrt{2x+1}}{\sqrt{x}}}=\lim_{x\to\infty}\frac{\frac{\sqrt{x}}{\sqrt{x}}+\frac{\sqrt[3]{x}}{\sqrt{x}}+\frac{\sqrt[4]{x}}{\sqrt{x}}}{\frac{\sqrt{2x+1}}{\sqrt{x}}}$$

$$\lim_{x\to\infty}\frac{1+\frac{x^{\frac{1}{3}}}{x^{\frac{1}{2}}}+\frac{x^{\frac{1}{4}}}{x^{\frac{1}{2}}}}{\sqrt{\left(\frac{2x}{x}+\frac{1}{x}\right)}}=\lim_{x\to\infty}\frac{1+\frac{1}{x^{\frac{1}{2}-\frac{1}{3}}}+\frac{1}{x^{\frac{1}{2}-\frac{1}{4}}}}{\sqrt{\left(\frac{2x}{x}+\frac{1}{x}\right)}}$$

$$\lim_{x\to\infty}\frac{1+\frac{1}{x^{\frac{1}{6}}}+\frac{1}{x^{\frac{1}{4}}}}{\sqrt{\left(2+\frac{1}{x}\right)}}=\lim_{x\to\infty}\frac{1+\frac{1}{\infty}+\frac{1}{\infty}}{\sqrt{\left(2+\frac{1}{\infty}\right)}}$$

$$\lim_{x\to\infty}\frac{1+0+0}{\sqrt{(2+0)}}=\lim_{x\to\infty}\frac{1}{\sqrt{2}}$$

$$\lim_{x\to\infty}\frac{\sqrt{x}+\sqrt[3]{x}+\sqrt[4]{x}}{\sqrt{2x+1}}=\frac{1}{\sqrt{2}}$$

53.- $\lim_{x\to\infty} \frac{\sqrt{x+\sqrt{x+\sqrt{x+3}}}}{\sqrt{x+3}}$

$$\lim_{x\to\infty} \frac{\sqrt{\infty+\sqrt{\infty+\sqrt{\infty+3}}}}{\sqrt{\infty+3}} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{\frac{\sqrt{x+\sqrt{x+\sqrt{x+3}}}}{\sqrt{x}}}{\frac{\sqrt{x+3}}{\sqrt{x}}} = \lim_{x\to\infty} \frac{\sqrt{\frac{x+\sqrt{x+\sqrt{x+3}}}{x}}}{\sqrt{\frac{x+3}{x}}}$$

$$\lim_{x\to\infty} \frac{\sqrt{\left(\frac{x}{x}+\frac{\sqrt{x+\sqrt{x+3}}}{x}\right)}}{\sqrt{\left(\frac{x}{x}+\frac{3}{x}\right)}} = \lim_{x\to\infty} \frac{\sqrt{\left(1+\sqrt{\frac{x+\sqrt{x+3}}{x}}\right)}}{\sqrt{\left(1+\frac{3}{x}\right)}}$$

$$\lim_{x\to\infty} \frac{\sqrt{\left(1+\sqrt{\left(\frac{x}{x}+\frac{\sqrt{x+3}}{x}\right)}\right)}}{\sqrt{\left(1+\frac{3}{x}\right)}} = \lim_{x\to\infty} \frac{\sqrt{\left(1+\sqrt{\left(1+\sqrt{\left(1+\frac{3}{x}\right)}\right)}\right)}}{\sqrt{\left(1+\frac{3}{x}\right)}}$$

$$\lim_{x\to\infty} \frac{\sqrt{\left(1+\sqrt{\left(1+\sqrt{\left(1+\frac{3}{\infty}\right)}\right)}\right)}}{\sqrt{\left(1+\frac{3}{\infty}\right)}} = \lim_{x\to\infty} \frac{\sqrt{\left(1+\sqrt{\left(1+\sqrt{(1+0)}\right)}\right)}}{\sqrt{(1+0)}}$$

$$\lim_{x\to\infty} \frac{\sqrt{(1+\sqrt{1})}}{\sqrt{(1+0)}} = \frac{\sqrt{1}}{\sqrt{1}} = 1$$

$$\lim_{x\to\infty} \frac{\sqrt{x+\sqrt{x+\sqrt{x+3}}}}{\sqrt{x+3}} = 1$$

54.- $\lim_{x\to\infty} \sqrt{x+\sqrt{x+\sqrt{x}}} - \sqrt{x}$

$$\lim_{x\to\infty} \sqrt{\infty+\sqrt{\infty+\sqrt{\infty}}} - \sqrt{\infty} = \infty - \infty$$

$$\lim_{x\to\infty} \sqrt{x+\sqrt{x+\sqrt{x}}} - \sqrt{x} \cdot \frac{\sqrt{x+\sqrt{x+\sqrt{x}}} + \sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}} + \sqrt{x}}$$

$$\lim_{x\to\infty} \frac{\sqrt{x+\sqrt{x+\sqrt{x}}}^{\,2} - \sqrt{x}^{\,2}}{\sqrt{x+\sqrt{x+\sqrt{x}}} + \sqrt{x}} = \lim_{x\to\infty} \frac{x+\sqrt{x+\sqrt{x}} - x}{\sqrt{x+\sqrt{x+\sqrt{x}}} + \sqrt{x}}$$

$$\lim_{x\to\infty} \frac{\sqrt{x+\sqrt{x}}}{\sqrt{x+\sqrt{x+\sqrt{x}}} + \sqrt{x}} = \frac{\frac{\sqrt{x+\sqrt{x}}}{\sqrt{x}}}{\frac{\sqrt{x+\sqrt{x+\sqrt{x}}} + \sqrt{x}}{\sqrt{x}}}$$

$$\lim_{x\to\infty}\frac{\sqrt{\left(\frac{x}{x}+\frac{\sqrt{x}}{x}\right)}}{\frac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x}}+\frac{\sqrt{x}}{\sqrt{x}}}=\lim_{x\to\infty}\frac{\sqrt{\left(1+\frac{\sqrt{x}}{x}\right)}}{\sqrt{\left(\frac{x}{x}+\frac{\sqrt{x+\sqrt{x}}}{x}\right)}+1}$$

$$\lim_{x\to\infty}\frac{\sqrt{\left(1+\frac{1}{x\cdot x^{\frac{1}{2}}}\right)}}{\sqrt{\left(1+\frac{\sqrt{x+\sqrt{x}}}{\sqrt{x^2}}\right)}+1}=\lim_{x\to\infty}\frac{\sqrt{\left(1+\frac{1}{x^{\frac{3}{2}}}\right)}}{\sqrt{\left(1+\sqrt{\left(\frac{1}{x}+\frac{\sqrt{x}}{\sqrt{x^2}^2}\right)}\right)}+1}$$

$$\lim_{x\to\infty}\frac{\sqrt{\left(1+\frac{1}{x^{\frac{3}{2}}}\right)}}{\sqrt{\left(1+\sqrt{\left(\frac{1}{x}+\sqrt{\frac{x}{x^4}}\right)}\right)}+1}=\lim_{x\to\infty}\frac{\sqrt{\left(1+\frac{1}{x^{\frac{3}{2}}}\right)}}{\sqrt{\left(1+\sqrt{\left(\frac{1}{x}+\sqrt{\frac{1}{x^3}}\right)}\right)}+1}$$

$$\lim_{x\to\infty}\frac{\sqrt{\left(1+\frac{1}{\infty^{\frac{3}{2}}}\right)}}{\sqrt{\left(1+\sqrt{\left(\frac{1}{\infty}+\sqrt{\frac{1}{\infty^3}}\right)}\right)}+1}=\lim_{x\to\infty}\frac{\sqrt{(1+0)}}{\sqrt{\left(1+\sqrt{(0+\sqrt{0})}\right)}+1}$$

$$\lim_{x\to\infty}\frac{\sqrt{1}}{\sqrt{(1)}+1}=\frac{1}{2}$$

$$\lim_{x\to\infty}\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}=\frac{1}{2}$$

55.- $\lim\limits_{x\to\infty}\frac{\sqrt[3]{8x^3+x^2}-\sqrt[3]{x^3+x^2}}{x}$

$$\lim_{x\to\infty}\frac{\sqrt[3]{8\cdot\infty^3+\infty^2}-\sqrt[3]{\infty^3+\infty^2}}{\infty}=\frac{0}{0}$$

$$\lim_{x\to\infty}\frac{\sqrt[3]{8x^3+x^2}-\sqrt[3]{x^3+x^2}}{\sqrt[3]{x}^3}=\lim_{x\to\infty}\frac{\sqrt[3]{8x^3+x^2}}{\sqrt[3]{x}^3}-\frac{\sqrt[3]{x^3+x^2}}{\sqrt[3]{x}^3}$$

$$\lim_{x\to\infty}\sqrt[3]{\frac{8x^3+x^2}{x^3}}-\sqrt[3]{\frac{x^3+x^2}{x^3}}=\sqrt[3]{\left(\frac{8x^3}{x^3}+\frac{x^2}{x^3}\right)}-\sqrt[3]{\left(\frac{x^3}{x^3}+\frac{x^2}{x^3}\right)}$$

$$\lim_{x\to\infty}\sqrt[3]{\left(8+\frac{1}{x^2}\right)}-\sqrt[3]{\left(1+\frac{1}{x^2}\right)}=\lim_{x\to\infty}\sqrt[3]{\left(8+\frac{1}{\infty}\right)}-\sqrt[3]{\left(1+\frac{1}{\infty}\right)}$$

$$\lim_{x\to\infty}\sqrt[3]{(8+0)}-\sqrt[3]{(1+0)}=\lim_{x\to\infty}\sqrt[3]{2^3}-\sqrt[3]{1}$$

$$\lim_{x\to\infty}2-1=1$$

$$\lim_{x\to\infty}\frac{\sqrt[3]{8x^3+x^2}-\sqrt[3]{x^3+x^2}}{x}=1$$

56.- $\lim\limits_{x\to\infty}x^2-\sqrt[3]{x^6-2x^4}$

$$\lim_{x\to\infty}\infty^2-\sqrt[3]{\infty^6-2\cdot\infty^4}=\infty-\infty$$

$$\lim_{x\to\infty}\left(x^2-\sqrt[3]{x^6-2x^4}\right)\cdot\frac{\left(x^4+x^2\cdot\sqrt[3]{x^6-2x^4}+\sqrt[3]{x^6-2x^4}^2\right)}{\left(x^4+x^2\cdot\sqrt[3]{x^6-2x^4}+\sqrt[3]{x^6-2x^4}^2\right)}$$

$$\lim_{x\to\infty}\left((x^2)^3-\sqrt[3]{x^6-2x^4}^3\right)\cdot\frac{1}{\left(x^4+x^2\cdot\sqrt[3]{x^6-2x^4}+\sqrt[3]{x^6-2x^4}^2\right)}$$

$$\lim_{x\to\infty}\frac{\left((x^2)^3-\sqrt[3]{x^6-2x^4}^3\right)}{\left(x^4+x^2\cdot\sqrt[3]{x^6-2x^4}+\sqrt[3]{x^6-2x^4}^2\right)}$$

$$\lim_{x\to\infty}\frac{(x^6-x^6+2x^4)}{\left(x^4+x^2\cdot\sqrt[3]{x^6-2x^4}+\sqrt[3]{x^6-2x^4}^2\right)}$$

$$\lim_{x\to\infty}\frac{2x^4}{\left(x^4+x^2\cdot\sqrt[3]{x^6-2x^4}+\sqrt[3]{x^6-2x^4}^2\right)}$$

$$\lim_{x\to\infty}\frac{\frac{2x^4}{x^4}}{\frac{x^4+x^2\cdot\sqrt[3]{x^6-2x^4}+\sqrt[3]{x^6-2x^4}^2}{x^4}}$$

$$\lim_{x\to\infty}\frac{2}{\frac{x^4}{x^4}+\frac{x^2\cdot\sqrt[3]{x^6-2x^4}}{x^4}+\frac{\sqrt[3]{x^6-2x^4}^2}{x^4}}$$

$$\lim_{x\to\infty}\frac{2}{1+\frac{\sqrt[3]{x^6-2x^4}}{x^2}+\left(\frac{\sqrt[3]{x^6-2x^4}}{x^2}\right)^2}$$

$$\lim_{x\to\infty} \frac{2}{1+\frac{\sqrt[3]{x^6-2x^4}}{\sqrt[3]{(x^2)^3}}+\left(\frac{\sqrt[3]{x^6-2x^4}}{\sqrt[3]{(x^2)^3}}\right)^2}$$

$$\lim_{x\to\infty} \frac{2}{1+\sqrt[3]{\frac{x^6-2x^4}{x^6}}+\left(\sqrt[3]{\frac{x^6-2x^4}{x^6}}\right)^2}$$

$$\lim_{x\to\infty} \frac{2}{1+\sqrt[3]{\left(\frac{x^6}{x^6}-\frac{2x^4}{x^6}\right)}+\sqrt[3]{\left(\frac{x^6}{x^6}-\frac{2x^4}{x^6}\right)}^2}$$

$$\lim_{x\to\infty} \frac{2}{1+\sqrt[3]{\left(1-\frac{2}{x^2}\right)}+\sqrt[3]{\left(1-\frac{2}{x^2}\right)}^2}$$

$$\lim_{x\to\infty} \frac{2}{1+\sqrt[3]{\left(1-\frac{2}{\infty}\right)}+\sqrt[3]{\left(1-\frac{2}{\infty}\right)}^2} = \lim_{x\to\infty} \frac{2}{1+\sqrt[3]{1}+\sqrt[3]{1}^2} = \frac{2}{3}$$

$$\lim_{x\to\infty} x^2-\sqrt[3]{x^6-2x^4} = \frac{2}{3}$$

57.- $\lim_{x\to\infty} \frac{\sqrt[3]{3x^3+2x}+x}{x-1}$

$$\lim_{x\to\infty}\frac{\sqrt[3]{3\cdot\infty^3+2\cdot\infty}\ +\ \infty}{\infty-1}=\frac{\infty}{\infty}$$

$$\lim_{x\to\infty}\frac{\sqrt[3]{3x^3+2x}\ +\ x}{x-1}\cdot\frac{\left(\left(\sqrt[3]{3x^3+2x}\right)^2-x\cdot\sqrt[3]{3x^3+2x}+x^2\right)}{\left(\left(\sqrt[3]{3x^3+2x}\right)^2-x\cdot\sqrt[3]{3x^3+2x}+x^2\right)}$$

$$\lim_{x\to\infty}\frac{\left(\sqrt[3]{3x^3+2x}\right)^3\ +\ x^3}{x-1}\cdot\frac{1}{\left(\left(\sqrt[3]{3x^3+2x}\right)^2-x\cdot\sqrt[3]{3x^3+2x}+x^2\right)}$$

$$\lim_{x\to\infty}\frac{3x^3+2x+\ x^3}{x-1}\cdot\frac{1}{\left(\left(\sqrt[3]{3x^3+2x}\right)^2-x\cdot\sqrt[3]{3x^3+2x}+x^2\right)}$$

$$\lim_{x\to\infty}\frac{4x^3+2x}{x-1}\cdot\frac{1}{\left(\left(\sqrt[3]{3x^3+2x}\right)^2-x\cdot\sqrt[3]{3x^3+2x}+x^2\right)}$$

$$\lim_{x\to\infty}\frac{\frac{4x^3+2x}{x^3}}{\frac{x-1}{x}}\cdot\frac{1}{\frac{\left(\sqrt[3]{3x^3+2x}\right)^2-x\cdot\sqrt[3]{3x^3+2x}+x^2}{x^2}}$$

$$\lim_{x\to\infty}\frac{\frac{4x^3}{x^3}+\frac{2x}{x^3}}{\frac{x}{x}-\frac{1}{x}}\cdot\frac{1}{\left(\frac{\left(\sqrt[3]{3x^3+2x}\right)^2}{x^2}-\frac{x\cdot\sqrt[3]{3x^3+2x}}{x^2}+\frac{x^2}{x^2}\right)}$$

$$\lim_{x\to\infty}\frac{4+\frac{2}{x^2}}{1-\frac{1}{x}}\cdot\frac{1}{\left(\left(\frac{\sqrt[3]{3x^3+2x}}{x}\right)^2-\frac{\sqrt[3]{3x^3+2x}}{x}+1\right)}$$

$$\lim_{x\to\infty} \frac{4+\frac{2}{x^2}}{1-\frac{1}{x}} \cdot \frac{1}{\left(\left(\frac{\sqrt[3]{3x^3+2x}}{\sqrt[3]{x^3}}\right)^2 - \frac{\sqrt[3]{3x^3+2x}}{\sqrt[3]{x^3}} + 1\right)}$$

$$\lim_{x\to\infty} \frac{4+\frac{2}{x^2}}{1-\frac{1}{x}} \cdot \frac{1}{\left(\sqrt[3]{\left(\frac{3x^3}{x^3}+\frac{2x}{x^3}\right)^2} - \sqrt[3]{\frac{3x^3}{x^3}+\frac{2x}{x^3}} + 1\right)}$$

$$\lim_{x\to\infty} \frac{4+\frac{2}{x^2}}{1-\frac{1}{x}} \cdot \frac{1}{\left(\sqrt[3]{\left(3+\frac{2}{x^2}\right)^2} - \sqrt[3]{3+\frac{2}{x^2}} + 1\right)}$$

$$\lim_{x\to\infty} \frac{4+\frac{2}{\infty}}{1-\frac{1}{\infty}} \cdot \frac{1}{\left(\sqrt[3]{\left(3+\frac{2}{\infty}\right)^2} - \sqrt[3]{3+\frac{2}{\infty}} + 1\right)}$$

$$\lim_{x\to\infty} \frac{4+0}{1-0} \cdot \frac{1}{\left(\sqrt[3]{(3+0)^2} - \sqrt[3]{3+0} + 1\right)} = \lim_{x\to\infty} \frac{4}{\left(\sqrt[3]{3^2} - \sqrt[3]{3} + 1\right)} \cdot \frac{\sqrt[3]{3}+1}{\sqrt[3]{3}+1}$$

$$\lim_{x\to\infty} \frac{4}{\left(\sqrt[3]{3^3}+1\right)} \cdot \frac{\sqrt[3]{3}+1}{1} = \lim_{x\to\infty} \frac{4}{4} \cdot \frac{\sqrt[3]{3}+1}{1} = \sqrt[3]{3}+1$$

$$\lim_{x\to\infty} \frac{\sqrt[3]{3x^3+2x} + x}{x-1} = \sqrt[3]{3}+1$$

1.6 Límites trigonométricos

Límites conocidos

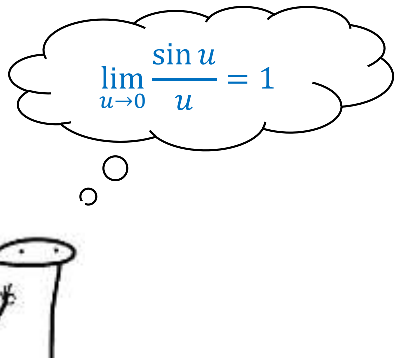
$$\lim_{u\to 0}\frac{\sin u}{u}=1$$

$$\lim_{u\to 0}\frac{1-\cos u}{u^2}=\frac{1}{2}$$

$$\lim_{u\to 0}\frac{1-\cos u}{u}=0$$

$$\lim_{u\to 0}\frac{\tan u}{u}=0$$

58.- $\lim\limits_{x\to 0}\frac{\sin 8x}{x}$



$$\lim_{x\to 0}\frac{\sin 8\cdot 0}{0}=\frac{\sin 0}{0}=\frac{0}{0}$$

$$\lim_{x\to 0}\frac{\sin 8x}{x}\cdot\frac{8}{8}$$

$$\lim_{x\to 0}\frac{\overbrace{\sin 8x}^{1}}{8x}\cdot 8=\lim_{x\to 0}8$$

$$\lim_{x\to 0}\frac{\sin 8x}{x}=8$$

59.- $\lim\limits_{x\to 0}\frac{8x-\sin 6x}{4x+5\sin 3x}$

$$\lim_{x\to 0}\frac{8\cdot 0-\sin 6\cdot 0}{4\cdot 0+5\sin 3\cdot 0}=\frac{0}{0}$$

$$\lim_{x\to 0}\frac{\frac{8x-\sin 6x}{x}}{\frac{4x+5\sin 3x}{x}}=\lim_{x\to 0}\frac{\frac{8x}{x}-\frac{\sin 6x}{x}}{\frac{4x}{x}+\frac{5\sin 3x}{x}}$$

$$\lim_{x\to 0}\frac{8-\frac{\sin 6x}{x}\cdot\frac{6}{6}}{4+\frac{5\sin 3x}{x}\cdot\frac{3}{3}}=\lim_{x\to 0}\frac{8-\frac{\sin 6x}{6x}\cdot\frac{6}{1}}{4+\frac{5\sin 3x}{3x}\cdot\frac{3}{1}}$$

$$\lim_{x\to 0}\frac{8-\overbrace{\frac{\sin 6x}{6x}}^{1}\cdot\frac{6}{1}}{4+5\underbrace{\frac{\sin 3x}{3x}}_{1}\cdot\frac{3}{1}}=\lim_{x\to 0}\frac{8-1\cdot 6}{4+5\cdot 1\cdot 3}$$

$$\lim_{x\to 0}\frac{8-6}{4+5\cdot 3}=\lim_{x\to 0}\frac{2}{4+8}=\lim_{x\to 0}\frac{2}{12}=\lim_{x\to 0}\frac{1}{6}$$

$$\lim_{x\to 0}\frac{8x-\sin 6x}{4x+5\sin 3x}=\frac{1}{6}$$

60.- $\lim_{x\to 0}\frac{1-\cos(\sin 4x)}{\sin^2(\sin 3x)}$

$$\lim_{x\to 0}\frac{1-\cos(\sin 4\cdot 0)}{\sin^2(\sin 3\cdot 0)}=\frac{0}{0}$$

$$\lim_{x\to 0}\frac{1-\cos(\sin 4x)}{\sin^2(\sin 3x)}=\lim_{x\to 0}\frac{\sin^2 4x\cdot\frac{1-\cos(\sin 4x)}{\sin^2 4x}}{\frac{\sin^2(\sin 3x)}{\sin^2 3x}\cdot\sin^2 3x}$$

$$\lim_{x\to 0}\frac{\sin^2 4x\cdot\overbrace{\frac{1-\cos(\sin 4x)}{\sin^2 4x}}^{\frac{1}{2}}}{\underbrace{\frac{\sin^2(\sin 3x)}{\sin^2 3x}}_{1}\cdot\sin^2 3x}=\lim_{x\to 0}\frac{\frac{(4x)^2}{(4x)^2}\cdot\sin^2 4x\cdot\frac{1}{2}}{\frac{(3x)^2}{(3x)^2}\cdot\sin^2 3x}$$

$$\lim_{x\to 0}\frac{(4x)^2\cdot\left(\frac{\sin 4x}{4x}\right)^2\cdot\frac{1}{2}}{(3x)^2\cdot\left(\frac{\sin 3x}{3x}\right)^2}=\lim_{x\to 0}\frac{(4x)^2\cdot\frac{1}{2}}{(3x)^2}$$

$$\lim_{x\to 0}\frac{4x^2\cdot\frac{1}{2}}{3x^2}=\lim_{x\to 0}\frac{2x^2}{3x^2}=\lim_{x\to 0}\frac{2}{3}$$

$$\lim_{x\to 0}\frac{1-\cos(\sin 4x)}{\sin^2(\sin 3x)}=\frac{2}{3}$$

61.-$\lim_{x\to\pi}\frac{1-\sin\frac{x}{2}}{\pi-x}$

$$\lim_{x\to\pi}\frac{1-\sin\frac{\pi}{2}}{\pi-\pi}=\frac{1-1}{\pi-\pi}=\frac{0}{0}$$

Cambio de variable sea

$x-\pi=\mathrm{h}\ \rightarrow\ \pi-\pi=0\quad \mathrm{h}\rightarrow 0$

$x=\mathrm{h}+\pi$

$$\lim_{\mathrm{h}\to 0}\frac{1-\sin\frac{\mathrm{h}+\pi}{2}}{\mathrm{h}}=\lim_{\mathrm{h}\to 0}\frac{1-\sin\left(\frac{\mathrm{h}}{2}+\frac{\pi}{2}\right)}{\mathrm{h}}$$

$$\sin(\alpha+\beta)=\sin\alpha\cdot\cos\beta+\sin\beta\cdot\cos\alpha$$

$$\lim_{h\to 0}\frac{1-\left(\sin\frac{h}{2}\cdot\cos\frac{\pi}{2}+\sin\frac{\pi}{2}\cdot\cos\frac{h}{2}\right)}{h}$$

$$\lim_{h\to 0}\frac{1-\left(\sin\frac{h}{2}\cdot\overbrace{\cos\frac{\pi}{2}}^{0}+\overbrace{\sin\frac{\pi}{2}}^{1}\cdot\cos\frac{h}{2}\right)}{h}$$

$$\cos\frac{\pi}{2}=\cos 90=0$$

$$\lim_{h\to 0}\frac{1-\left(1\cdot\cos\frac{h}{2}\right)}{h}=\frac{1-\cos\frac{h}{2}}{h}$$

$$\lim_{h\to 0}\frac{1-\cos\frac{h}{2}}{2\cdot\frac{h}{2}}=\frac{1}{2}\cdot\frac{1-\cos\frac{h}{2}}{\frac{h}{2}}=\frac{1}{2}\cdot 0=0$$

$$\lim_{x\to\pi}\frac{1-\sin\frac{x}{2}}{\pi-x}=0$$

62.- $\lim\limits_{x\to 0}\dfrac{\cos x-\cos 3x}{x^2}$

$$\lim_{x\to 0}\frac{\cos 0-\cos 3\cdot 0}{0}=\frac{1-1}{0}=\frac{0}{0}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\cdot\sin\frac{\alpha-\beta}{2}$$

$$\lim_{x\to 0}\frac{\cos x - \cos 3x}{x^2} = \lim_{x\to 0}\frac{-2\sin\frac{x+3x}{2}\cdot\sin\frac{x-3x}{2}}{x^2}$$

$$\lim_{x\to 0}\frac{-2\sin\frac{4x}{2}\cdot\sin\frac{-2x}{2}}{x^2} = \lim_{x\to 0}\frac{-2\sin 2x\cdot\sin -x}{x\cdot x}$$

$$\lim_{x\to 0}\frac{-2\sin 2x}{x}\cdot\frac{\sin -x}{x} = \lim_{x\to 0}\frac{-2\sin 2x}{x}\cdot\frac{2}{2}\cdot\frac{\sin -x}{x}\cdot\frac{-1}{-1}$$

$$\lim_{x\to 0} -2\cdot\frac{\sin 2x}{2x}\cdot\frac{2}{1}\cdot\frac{\sin -x}{-x}\cdot\frac{-1}{1} = \lim_{x\to 0} -2\cdot 1\cdot\frac{2}{1}\cdot 1\cdot\frac{-1}{1} = \lim_{x\to 0} 4$$

$$\lim_{x\to 0}\frac{\cos x - \cos 3x}{x^2} = 4$$

63.-$\lim_{x\to 0}\frac{\mathrm{x}-\sin 2x}{x+sin3x}$

$$\lim_{x\to 0}\frac{0-\sin 2\cdot 0}{0+sin3\cdot 0} = \frac{0}{0}$$

$$\lim_{x\to 0}\frac{\mathrm{x}-\sin 2x}{x+sin3x} = \lim_{x\to 0}\frac{\mathrm{x}\left(1-\frac{\sin 2x}{x}\right)}{\mathrm{x}\left(1-\frac{\sin 3x}{x}\right)}$$

$$\lim_{x\to 0}\frac{x\left(1-\frac{\sin 2x}{x}\cdot\frac{2}{2}\right)}{x\left(1-\frac{\sin 3x}{x}\cdot\frac{3}{3}\right)}=\lim_{x\to 0}\frac{\left(1-\frac{\sin 2x}{2x}\cdot\frac{2}{1}\right)}{\left(1-\frac{\sin 3x}{3x}\cdot\frac{3}{1}\right)}$$

$$\lim_{x\to 0}\frac{\left(1-1\cdot\frac{2}{1}\right)}{\left(1-1\cdot\frac{3}{1}\right)}=\frac{-1}{-2}=\frac{1}{2}$$

$$\lim_{x\to 0}\frac{x-\sin 2x}{x+sin3x}=\frac{1}{2}$$

64.- $\lim_{x\to\frac{\pi}{4}}\frac{\cos x-\sin x}{\cos 2x}$

$$\lim_{x\to\frac{\pi}{4}}\frac{\cos\frac{\pi}{4}-\sin\frac{\pi}{4}}{\cos 2\cdot\frac{\pi}{4}}=\frac{\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}}{0}=\frac{0}{0}$$

$$\cos 2\alpha = cos^2\alpha - sin^2\alpha$$

$$\lim_{x\to\frac{\pi}{4}}\frac{\cos x-\sin x}{cos^2\alpha - sin^2\alpha}=\lim_{x\to\frac{\pi}{4}}\frac{(\cos x-\sin x)}{(\cos x-\sin x)(\cos x+\sin x)}$$

$$\lim_{x\to\frac{\pi}{4}}\frac{1}{(\cos x+\sin x)}=\lim_{x\to\frac{\pi}{4}}\frac{1}{\left(\cos\frac{\pi}{4}+\sin\frac{\pi}{4}\right)}=\lim_{x\to\frac{\pi}{4}}\frac{1}{\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\right)}$$

$$\lim_{x\to\frac{\pi}{4}}\frac{1}{\left(2\frac{\sqrt{2}}{2}\right)}=\frac{1}{\sqrt{2}}$$

$$\lim_{x\to\pi}\frac{\cos x-\sin x}{\cos 2x}=\frac{1}{\sqrt{2}}$$

65.- $\lim\limits_{x\to\frac{\pi}{4}}\frac{\sin x-\cos x}{1-\tan x}$

$$\lim_{x\to\frac{\pi}{4}}\frac{\sin\frac{\pi}{4}-\cos\frac{\pi}{4}}{1-\tan\frac{\pi}{4}}=\frac{\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}}{1-1}=\frac{0}{0}$$

$$\lim_{x\to\frac{\pi}{4}}\frac{\sin x-\cos x}{1-\tan x}=\lim_{x\to\frac{\pi}{4}}\frac{\sin x-\cos x}{1-\frac{\sin x}{\cos x}}$$

$$\lim_{x\to\frac{\pi}{4}}\frac{\sin x-\cos x}{\frac{\cos x\ -\sin x}{\cos x}}=\lim_{x\to\frac{\pi}{4}}\frac{\sin x-\cos x}{-(\sin x-\cos x)}\cdot\frac{\cos x}{1}$$

$$\lim_{x\to\frac{\pi}{4}}-\cos x=\lim_{x\to\frac{\pi}{4}}-\cos\frac{\pi}{4}=-\frac{\sqrt{2}}{2}$$

$$\lim_{x\to\frac{\pi}{4}}\frac{\sin x-\cos x}{1-\tan x}=-\frac{\sqrt{2}}{2}$$

66.- $\lim\limits_{x\to a}\frac{\cos x-\cos a}{x-a}$

$$\lim_{x\to a}\frac{\cos a-\cos a}{a-a}=\frac{0}{0}$$

$$\cos\alpha-\cos\beta=-2\sin\frac{\alpha+\beta}{2}\cdot\sin\frac{\alpha-\beta}{2}$$

$$\lim_{x\to a}\frac{\cos x-\cos a}{x-a}=\lim_{x\to a}\frac{-2\sin\frac{x+a}{2}\cdot\sin\frac{x-a}{2}}{x-a}$$

$$\lim_{x\to a}\frac{-2\sin\frac{x+a}{2}}{1}\cdot\frac{\sin\frac{x-a}{2}}{x-a}=\lim_{x\to a}-2\sin\frac{x+a}{2}\cdot\frac{\sin\frac{x-a}{2}}{\frac{x-a}{2}\cdot 2}$$

$$\lim_{x\to a}-2\sin\frac{x+a}{2}\cdot\frac{\sin\frac{x-a}{2}}{\frac{x-a}{2}}\cdot\frac{1}{2}=\lim_{x\to a}-2\sin\frac{x+a}{2}\cdot 1\cdot\frac{1}{2}$$

$$\lim_{x\to a}-\sin\frac{x+a}{2}=\lim_{x\to a}-\sin\frac{a+a}{2}=\lim_{x\to a}\sin\frac{2a}{2}=-\sin a$$

$$\lim_{x\to a}\frac{\cos x-\cos a}{x-a}=-\sin a$$

67.- $\lim\limits_{x\to 0}\frac{1-2\cos x+\cos 2x}{x^2}$

$$\lim_{x\to 0}\frac{1-2\cos 0+\cos 2\cdot 0}{0^2}=\frac{0}{0}$$

$\cos(2\theta)=\cos^2\theta-\sin^2\theta$

$$\lim_{x\to 0}\frac{1-2\cos x+\cos 2x}{x^2}=\lim_{x\to 0}\frac{1-2\cos x+\cos^2 x-\sin^2 x}{x^2}$$

$\sin^2\theta+\cos^2\theta=1$

$$\lim_{x\to 0}\frac{\sin^2 x+\cos^2 x-2\cos x+\cos^2 x-\sin^2 x}{x^2}$$

$$\lim_{x\to 0}\frac{2\cos^2 x-2\cos x}{x^2}=\lim_{x\to 0}\frac{2\cos x\,(\cos x-1)}{x^2}$$

$$\lim_{x\to 0}-\frac{2\cos x\,(1-\cos x)}{x^2}=\lim_{x\to 0}-2\cos x\cdot\frac{1-\cos x}{x^2}$$

$$\lim_{x\to 0}-2\cos x\cdot\lim_{x\to 0}\frac{1-\cos x}{x^2}=\lim_{x\to 0}-2\cos 0\cdot\frac{1}{2}=-1$$

$$\lim_{x\to 0}\frac{1-2\cos x+\cos 2x}{x^2}=-1$$

68.- $\lim_{x\to -\frac{\pi}{2}}\frac{(\pi+2x)\cos\left(\frac{3\pi}{2}+3x\right)}{\sin\left(3\frac{\pi}{2}+3x\right)}$

$$\lim_{x\to -\frac{\pi}{2}}\frac{\left(\pi+2\cdot-\frac{\pi}{2}\right)\cos\left(\frac{3\pi}{2}+3\cdot-\frac{\pi}{2}\right)}{\sin\left(3\frac{\pi}{2}+3\cdot-\frac{\pi}{2}\right)}=\frac{(\pi-\pi)\cos\left(\frac{3\pi}{2}-\frac{3\pi}{2}\right)}{\sin\left(\frac{3\pi}{2}-\frac{3\pi}{2}\right)}=\frac{0}{0}$$

$$\cos(\alpha+\theta)=\cos\alpha\cdot\cos\theta-\sin\alpha\cdot\sin\theta$$

$$\lim_{x\to -\frac{\pi}{2}}\frac{(\pi+2x)\cos\left(\frac{3\pi}{2}+3x\right)}{\sin\left(3\frac{\pi}{2}+3x\right)}=\lim_{x\to -\frac{\pi}{2}}\frac{(\pi+2x)(\cos\frac{3\pi}{2}\cdot\cos 3x-\sin\frac{3\pi}{2}\cdot\sin 3x)}{\sin\left(3\frac{\pi}{2}+3x\right)}$$

$$\lim_{x\to-\frac{\pi}{2}} \frac{(\pi+2x)(\cos\frac{3\pi}{2}\cdot\cos 3x-\sin\frac{3\pi}{2}\cdot\sin 3x)}{\sin\left(3\frac{\pi}{2}+3x\right)}$$

$$\lim_{x\to-\frac{\pi}{2}} \frac{(\pi+2x)(0\cdot\cos 3x-(-1)\cdot\sin 3x)}{\sin\left(3\frac{\pi}{2}+3x\right)}=\lim_{x\to-\frac{\pi}{2}} \frac{(\pi+2x)(\sin 3x)}{\sin\left(3\frac{\pi}{2}+3x\right)}$$

$$\sin(\alpha+\theta)=\sin\alpha\cdot\cos\theta+\sin\theta\cdot\cos\alpha$$

$$\lim_{x\to-\frac{\pi}{2}} \frac{(\pi+2x)(\sin 3x)}{\sin\left(3\frac{\pi}{2}+3x\right)}=\lim_{x\to-\frac{\pi}{2}} \frac{(\pi+2x)(\sin 3x)}{\sin 3\frac{\pi}{2}\cdot\cos 3x+\sin 3x\cdot\cos 3\frac{\pi}{2}}$$

$$\lim_{x\to-\frac{\pi}{2}} \frac{(\pi+2x)(\sin 3x)}{-1\cdot\cos 3x+\sin 3x\cdot 0}=\lim_{x\to-\frac{\pi}{2}} -\frac{(\pi+2x)(\sin 3x)}{\cos 3x}$$

Cambio de variable sea:

$$x+\frac{\pi}{2}=\mathrm{h}\quad,-\frac{\pi}{2}+\frac{\pi}{2}=0\ \Rightarrow \mathrm{h}\to 0$$

$$x=\mathrm{h}-\frac{\pi}{2}$$

$$\lim_{x\to 0} -\frac{\left(\pi+2\left(\mathrm{h}-\frac{\pi}{2}\right)\right)(\sin 3\cdot\left(\mathrm{h}-\frac{\pi}{2}\right))}{\cos 3\cdot\left(\mathrm{h}-\frac{\pi}{2}\right)}$$

$$\lim_{x\to 0} -\frac{(\pi+2\mathrm{h}-\pi)\left(\sin\left(3\mathrm{h}-3\frac{\pi}{2}\right)\right)}{\cos\left(3\mathrm{h}-3\frac{\pi}{2}\right)}=\lim_{x\to 0} -\frac{2\mathrm{h}\left(\sin\left(3\mathrm{h}-3\frac{\pi}{2}\right)\right)}{\cos\left(3\mathrm{h}-3\frac{\pi}{2}\right)}$$

$$\sin(\alpha-\theta)=\sin\alpha\cdot\cos\theta-\sin\theta\cdot\cos\alpha$$

$$\cos(\alpha-\theta)=\cos\alpha\cdot\cos\theta+\sin\alpha\cdot\sin\theta$$

$$\lim_{x\to 0}-\frac{2\text{h}\left(\sin 3\text{h}\cdot\cos 3\frac{\pi}{2}-\sin 3\frac{\pi}{2}\cdot\cos 3\text{h}\right)}{\cos 3\text{h}\cdot\cos 3\frac{\pi}{2}+\sin 3\text{h}\cdot\sin 3\frac{\pi}{2}}$$

$$\lim_{x\to 0}-\frac{2\text{h}(\sin 3\text{h}\cdot 0-(-1)\cdot\cos 3\text{h})}{\cos 3\text{h}\cdot 0+\sin 3\text{h}\cdot(-1)}=\lim_{x\to 0}-\frac{2\text{h}(\cos 3\text{h})}{-\sin 3\text{h}}$$

$$\lim_{x\to 0}\frac{2\text{h}(\cos 3\text{h})}{\sin 3\text{h}}=\lim_{x\to 0}\frac{2(\cos 3\text{h})}{\frac{\sin 3\text{h}}{\text{h}}\cdot\frac{3}{3}}$$

$$\lim_{x\to 0}\frac{2(\cos 3\text{h})}{\frac{\sin 3\text{h}}{3\text{h}}\cdot 3}=\frac{2\cdot\lim_{x\to 0}(\cos 3\text{h})}{3\cdot\lim_{x\to 0}\frac{\sin 3\text{h}}{3\text{h}}}$$

$$\frac{2\cdot\lim_{x\to 0}(\cos 3\cdot 0)}{3\cdot 1}=\frac{2\cdot\lim_{x\to 0}(1)}{3\cdot 1}=\frac{2}{3}$$

$$\lim_{x\to-\frac{\pi}{2}}\frac{(\pi+2x)\cos\left(\frac{3\pi}{2}+3x\right)}{\sin\left(3\frac{\pi}{2}+3x\right)}=\frac{2}{3}$$

69.-$\lim\limits_{x\to 0}\left(\sqrt[3]{\frac{\cos(mx)-\cos(nx)}{x^2}}\right)$

$$\cos\alpha-\cos\beta=-2\sin\frac{\alpha+\beta}{2}\cdot\sin\frac{\alpha-\beta}{2}$$

$$\sqrt[3]{\left(\lim_{x\to 0}\frac{-2\sin\frac{mx+nx}{2}\cdot\sin\frac{mx-nx}{2}}{x^2}\right)}$$

$$\sqrt[3]{\left(\lim_{x\to 0}\frac{-2\sin\frac{mx+nx}{2}\cdot\sin\frac{mx-nx}{2}}{x^2}\cdot\frac{\frac{mx+nx}{2}}{\frac{mx+nx}{2}}\cdot\frac{\frac{mx-nx}{2}}{\frac{mx-nx}{2}}\right)}$$

$$\sqrt[3]{\left(\lim_{x\to 0}-2\frac{\sin\frac{mx+nx}{2}}{\frac{mx+nx}{2}}\cdot\frac{\sin\frac{mx-nx}{2}}{\frac{mx-nx}{2}}\cdot\frac{\frac{mx+nx}{2}}{x^2}\cdot\frac{\frac{mx-nx}{2}}{1}\right)}$$

$$\sqrt[3]{\left(\lim_{x\to 0}-2\cdot 1\cdot 1\cdot\frac{\frac{mx+nx}{2}}{x^2}\cdot\frac{\frac{mx-nx}{2}}{1}\right)}=\sqrt[3]{\left(\lim_{x\to 0}-2\cdot\frac{\frac{(mx)^2-(nx)^2}{4}}{x^2}\right)}$$

$$\sqrt[3]{\left(\lim_{x\to 0}-2\cdot\frac{(mx)^2-(nx)^2}{4x^2}\right)}=\sqrt[3]{\left(\lim_{x\to 0}-2\cdot\frac{x^2(m^2-n^2)}{4x^2}\right)}$$

$$\sqrt[3]{\left(\lim_{x\to 0} - 2\cdot\frac{(m^2-n^2)}{4}\right)} = \sqrt[3]{\left(\frac{(n^2-m^2)}{2}\right)}$$

$$\lim_{x\to 0}\left(\sqrt[3]{\frac{\cos(mx)-\cos(nx)}{x^2}}\right) = \sqrt[3]{\left(\frac{(n^2-m^2)}{2}\right)}$$

70.- $\lim_{x\to 1}\left(\frac{x^2-x}{\sin(x-1)} + \frac{\sqrt{x}-1}{\sin(x-1)}\right)$

$$\lim_{x\to 1}\left(\frac{1^2-1}{\sin(1-1)} + \frac{\sqrt{1}-1}{\sin(1-1)}\right) = \frac{0}{0}$$

$$\lim_{x\to 1}\left(\frac{x^2-x}{\sin(x-1)} + \frac{\sqrt{x}-1}{\sin(x-1)}\right)$$

$$\lim_{x\to 1}\left(\frac{x(x-1)}{\sin(x-1)} + \frac{\sqrt{x}-1}{\sin(x-1)}\cdot\frac{\sqrt{x}+1}{\sqrt{x}+1}\right)$$

$$\lim_{x\to 1}\left(\frac{x}{\frac{\sin(x-1)}{(x-1)}} + \frac{\sqrt{x}^2-1}{\sin(x-1)}\cdot\frac{1}{\sqrt{x}+1}\right)$$

$$\lim_{x\to 1}\left(\frac{x}{\frac{\sin(x-1)}{(x-1)}} + \frac{(x-1)}{\sin(x-1)}\cdot\frac{1}{\sqrt{x}+1}\right)$$

$$\lim_{x\to 1}\left(\frac{x}{\frac{\sin(x-1)}{(x-1)}}+\frac{1}{\frac{\sin(x-1)}{(x-1)}}\cdot\frac{1}{\sqrt{x}+1}\right)$$

$$\lim_{x\to 1}\left(\frac{x}{1}+\frac{1}{1}\cdot\frac{1}{\sqrt{x}+1}\right)=\lim_{x\to 1}\left(x+\frac{1}{\sqrt{x}+1}\right)=\lim_{x\to 1}\left(1+\frac{1}{\sqrt{1}+1}\right)$$

$$\lim_{x\to 1}\left(1+\frac{1}{2}\right)=\lim_{x\to 1}\left(\frac{3}{2}\right)=\frac{3}{2}$$

$$\lim_{x\to 1}\left(\frac{x^2-x}{\sin(x-1)}+\frac{\sqrt{x}-1}{\sin(x-1)}\right)=\frac{3}{2}$$

1.7 Límites exponenciales y logarítmicos

Límites conocidos

$$\lim_{u\to\infty}\left(1+\frac{1}{u}\right)^{u}=e$$

$$\lim_{u\to 0}(1+u)^{\frac{1}{u}}=e$$

$$\lim_{u\to 0}\frac{a^u-1}{u}=\ln a$$

$$\lim_{u\to 0}\frac{e^u-1}{u}=1$$

$$\lim_{x\to a}\big(f(x)\big)^{g(x)}=\lim_{x\to a}\left[\left(1+f_{(x)}\right)^{\frac{1}{f(x)}}\right]^{f(x)\cdot g(x)}=e^{\lim_{x\to a}f(x)\cdot g(x)}$$

71.- $\lim_{x\to\infty}\left(\frac{x^3+2x+3}{x^3+4}\right)^{x^2+2}$

$$\lim_{x\to\infty}\left(\frac{\infty^3+2\cdot\infty+3}{\infty^3+4}\right)^{\infty^2+2}=\left(\frac{\infty}{\infty}\right)^{\infty}$$

$$\lim_{x\to\infty}\left(1+\frac{x^3+2x+3}{x^3+4}-1\right)^{x^2+2}$$

$$\lim_{x\to\infty}\left(1+\frac{x^3+2x+3-x^3-4}{x^3+4}\right)^{x^2+2}$$

$$\lim_{x\to a}\left[\left(1+f_{(x)}\right)^{\frac{1}{f(x)}}\right]^{f(x)\cdot g(x)}=e^{\lim_{x\to a}f(x)\cdot g(x)}$$

$$\lim_{x\to\infty}\left(1+\frac{2x-1}{x^3+4}\right)^{x^2+2}=\lim_{x\to\infty}\left[\left(1+\frac{2x-1}{x^3+4}\right)^{\frac{1}{\frac{2x-1}{x^3+4}}}\right]^{\frac{2x-1}{x^3+4}\cdot(x^2+2)}$$

$$e^{\lim_{x\to\infty}\frac{2x-1}{x^3+4}\cdot(x^2+2)}$$

$$e^{\lim_{x\to\infty}\frac{2x-1}{x^3+4}\cdot(x^2+2)}=e^{\lim_{x\to\infty}\frac{2x^3-x^2+4x-2}{x^3+4}}=e^{\lim_{x\to\infty}\frac{\frac{2x^3-x^2+4x-2}{x^3}}{\frac{x^3+4}{x^3}}}$$

$$e^{\lim_{x\to\infty}\frac{\frac{2x^3}{x^3}-\frac{x^2}{x^3}+\frac{4x}{x^3}-\frac{2}{x^3}}{\frac{x^3}{x^3}+\frac{4}{x^3}}}=e^{\lim_{x\to\infty}\frac{2-\frac{1}{x}+\frac{4}{x^2}-\frac{2}{x^3}}{1+\frac{4}{x^3}}}=e^{\lim_{x\to\infty}\frac{2-\frac{1}{\infty}+\frac{4}{\infty^2}-\frac{2}{\infty^3}}{1+\frac{4}{\infty^3}}}$$

$$e^{\lim\limits_{x\to\infty}\frac{2-0+0-0}{1+0}} = e^{\lim\limits_{x\to\infty} 2} = e^2$$

$$\lim_{x\to\infty}\left(\frac{x^3+2x+3}{x^3+4}\right)^{x^2+2} = e^2$$

72.- $\lim\limits_{x\to\infty}\left(\frac{3x-4}{3x+2}\right)^{\frac{x+1}{3}}$

$$\lim_{x\to\infty}\left(\frac{3\cdot\infty-4}{3\cdot\infty+2}\right)^{\frac{\infty+1}{3}} = \left(\frac{\infty}{\infty}\right)^{\infty}$$

$$\lim_{x\to a}\left[\left(1+f_{(x)}\right)^{\frac{1}{f(x)}}\right]^{f(x)\cdot g(x)} = e^{\lim\limits_{x\to a} f(x)\cdot g(x)}$$

$$\lim_{x\to\infty}\left(\frac{3x-4}{3x+2}\right)^{\frac{x+1}{3}} = \lim_{x\to\infty}\left(1+\frac{3x-4}{3x+2}-1\right)^{\frac{x+1}{3}}$$

$$\lim_{x\to\infty}\left(1+\frac{3x-4-3x-2}{3x+2}\right)^{\frac{x+1}{3}} = \lim_{x\to\infty}\left(1+\frac{-6}{3x+2}\right)^{\frac{x+1}{3}}$$

$$\lim_{x\to\infty}\left[\left(1+\frac{-6}{3x+2}\right)^{\frac{1}{\frac{-6}{3x+2}}}\right]^{\frac{-6}{3x+2}\cdot\frac{x+1}{3}} = e^{\lim\limits_{x\to\infty} -2\frac{1}{3x+2}\cdot\frac{x+1}{1}}$$

$$e^{-2\lim\limits_{x\to\infty}\frac{x+1}{3x+2}} = e^{-2\lim\limits_{x\to\infty}\frac{\frac{x+1}{x}}{\frac{3x+2}{x}}} = e^{-2\lim\limits_{x\to\infty}\frac{\frac{x}{x}+\frac{1}{x}}{\frac{3x}{x}+\frac{2}{x}}}$$

$$e^{-2\lim\limits_{x\to\infty}\frac{1+\frac{1}{x}}{3+\frac{2}{x}}} = e^{-2\lim\limits_{x\to\infty}\frac{1+\frac{1}{\infty}}{3+\frac{2}{\infty}}} = e^{-2\lim\limits_{x\to\infty}\frac{1+0}{3+0}}$$

$$e^{-2\lim\limits_{x\to\infty}\frac{1}{3}} = e^{-\frac{2}{3}}$$

$$\lim_{x\to\infty}\left(\frac{3x-4}{3x+2}\right)^{\frac{x+1}{3}} = e^{-\frac{2}{3}}$$

73.-$\lim\limits_{x\to 0}(\cos x+\sin x)^{\frac{1}{x}}$

$$\lim_{x\to 0}(\cos 0+\sin 0)^{\frac{1}{0}} = 1^{\infty}$$

$$\lim_{x\to 0}(1+\cos x+\sin x\ -1)^{\frac{1}{x}}$$

$$\lim_{x\to a}\left[\left(1+f_{(x)}\right)^{\frac{1}{f_{(x)}}}\right]^{f_{(x)}\cdot g_{(x)}} = e^{\lim\limits_{x\to a} f_{(x)}\cdot g_{(x)}}$$

$$\lim_{x\to 0}\left[(1+\cos x+\sin x\ -1)^{\frac{1}{\cos x+\sin x\,-1}}\right]^{(\cos x+\sin x\,-1)\cdot\frac{1}{x}}$$

$$e^{\lim\limits_{x\to 0}(\cos x+\sin x\,-1)\cdot\frac{1}{x}}$$

$$e^{\lim\limits_{x\to 0}\left(\frac{\cos x-1}{x}+\frac{\sin x}{x}\right)} = e^{\lim\limits_{x\to 0}(0+1)} = e^{1}$$

$$\lim_{x\to 0}(\cos x+\sin x)^{\frac{1}{x}} = e$$

74.- $\lim_{x\to 0}\left(\frac{x^2-2x+3}{x^2-3x+2}\right)^{\frac{\sin x}{x}}$

$$\lim_{x\to 0}\left(\frac{0^2-2\cdot 0+3}{0^2-3\cdot 0+2}\right)^{\frac{\sin 0}{0}} = (1)^{\infty}$$

$$\lim_{x\to 0}\left(1+\frac{x^2-2x+3}{x^2-3x+2}-1\right)^{\frac{\sin x}{x}}$$

$$\lim_{x\to 0}\left(1+\frac{x^2-2x+3-x^2+3x-2}{x^2-3x+2}\right)^{\frac{\sin x}{x}}$$

$$\lim_{x\to 0}\left(1+\frac{x+1}{x^2-3x+2}\right)^{\frac{\sin x}{x}}$$

$$\lim_{x\to a}\left[\left(1+f_{(x)}\right)^{\frac{1}{f_{(x)}}}\right]^{f_{(x)}\cdot g_{(x)}} = e^{\lim_{x\to a} f_{(x)}\cdot g_{(x)}}$$

$$\lim_{x\to 0}\left[\left(1+\frac{x+1}{x^2-3x+2}\right)^{\frac{1}{\frac{x+1}{x^2-3x+2}}}\right]^{\frac{x+1}{x^2-3x+2}\cdot\frac{\sin x}{x}}$$

$$e^{\lim_{x\to 0}\frac{x+1}{x^2-3x+2}\cdot\frac{\sin x}{x}}$$

$$e^{\lim_{x\to 0}\frac{x+1}{x^2-3x+2}\cdot\lim_{x\to 0}\frac{\sin x}{x}} = e^{\lim_{x\to 0}\frac{0+1}{0-3\cdot 0+2}\cdot 1} = e^{\frac{1}{2}}$$

$$\lim_{x\to 0}\left(\frac{x^2-2x+3}{x^2-3x+2}\right)^{\frac{\sin x}{x}}=\sqrt{e}$$

75.- $\lim_{x\to\infty}\left(\frac{x^2-1}{x^2+1}\right)^{\frac{x-1}{x+1}}$

$$\lim_{x\to\infty}\left(\frac{\infty^2-1}{\infty^2+1}\right)^{\frac{\infty-1}{\infty+1}}=\infty^{\infty}$$

$$\lim_{x\to a}[f(x)]^{g(x)}=M^N$$

$$\lim_{x\to a} f(x)=M\,;\lim_{x\to a} g(x)=N$$

$$M=\lim_{x\to\infty}\frac{x^2-1}{x^2+1}=\lim_{x\to\infty}\frac{\frac{x^2-1}{x^2}}{\frac{x^2-1}{x^2}}=\lim_{x\to\infty}\frac{\frac{x^2}{x^2}-\frac{1}{x^2}}{\frac{x^2}{x^2}+\frac{1}{x^2}}$$

$$\lim_{x\to\infty}\frac{1-\frac{1}{\infty}}{1+\frac{1}{\infty}}=\lim_{x\to\infty}\frac{1-0}{1+0}=1$$

N = $\lim_{x\to\infty}\frac{x-1}{x+1}=\lim_{x\to\infty}\frac{\frac{x-1}{x}}{\frac{x-1}{x}}=\lim_{x\to\infty}\frac{\frac{x}{x}-\frac{1}{x}}{\frac{x}{x}+\frac{1}{x}}$

$$\lim_{x\to\infty}\frac{1-\frac{1}{\infty}}{1+\infty}=\lim_{x\to\infty}\frac{1-0}{1+0}=1$$

$$M^N=1^1$$

$$\lim_{x\to\infty}\left(\frac{x^2-1}{x^2+1}\right)^{\frac{x-1}{x+1}}=1$$

76.$-\lim_{x\to 0}\frac{9^x-7^x}{8^x-6^x}$

$$\lim_{x\to 0}\frac{9^0-7^0}{8^0-6^0}=\frac{0}{0}$$

$$\lim_{x\to 0}\frac{(9^x-1)-(7^x-1)}{(8^x-1)-(6^x-1)}$$

$$\lim_{x\to 0}\frac{\frac{(9^x-1)-(7^x-1)}{x}}{\frac{(8^x-1)-(6^x-1)}{x}}=\lim_{x\to 0}\frac{\frac{9^x-1}{x}-\frac{7^x-1}{x}}{\frac{8^x-1}{x}-\frac{6^x-1}{x}}$$

$$\lim_{x\to 0}\frac{\ln 9-\ln 7}{\ln 8-\ln 6}=\lim_{x\to 0}\frac{\ln\frac{9}{7}}{\ln\frac{8}{6}}=\lim_{x\to 0}\frac{\ln\frac{9}{7}}{\ln\frac{4}{3}}$$

$$\lim_{x\to 0}\frac{9^x-7^x}{8^x-6^x}=\frac{\ln\frac{9}{7}}{\ln\frac{4}{3}}$$

77.-$\lim_{x\to 0}(\mathrm{e}^x+x)^{\frac{m}{x}}$

$$\lim_{x\to 0}(\mathrm{e}^0+0)^{\frac{m}{0}}=1^\infty$$

$$\lim_{x\to 0}(\mathrm{e}^x+x)^{\frac{m}{x}}=\lim_{x\to 0}(1+\mathrm{e}^x+x-1)^{\frac{m}{x}}$$

$$\lim_{x\to 0}\left[(1+e^x+x-1)^{\frac{1}{(e^x+x-1)}}\right]^{(e^x+x-1)\cdot\frac{m}{x}} = e^{\lim_{x\to 0}(e^x+x-1)\cdot\frac{m}{x}}$$

$$e^{\lim_{x\to 0}\left(\frac{e^x+x-1}{x}\right)\cdot m} = e^{\lim_{x\to 0}\left(\frac{e^x-1}{x}+\frac{x}{x}\right)\cdot m}$$

$$e^{\lim_{x\to 0}(1+1)\cdot m} = e^{2m}$$

$$\lim_{x\to 0}(e^x+x)^{\frac{m}{x}} = e^{2m}$$

78.-$\lim_{x\to 0}\frac{\sin 3x-\sin x}{In(1+x)}$

$$\lim_{x\to 0}\frac{\sin 3\cdot 0-\sin 0}{In(1+0)} = \frac{0}{0}$$

$$\lim_{x\to 0}\frac{\frac{\sin 3x-\sin x}{x}}{\frac{In(1+x)}{x}} = \lim_{x\to 0}\frac{\frac{\sin 3x}{x}-\frac{\sin x}{x}}{\frac{1}{x}\,In(1+x)}$$

$$\lim_{x\to 0}\frac{\frac{\sin 3x}{3x}\cdot 3-\frac{\sin x}{x}}{In(1+x)^{\frac{1}{x}}} = \frac{3\lim_{x\to 0}\frac{\sin 3x}{3x}-\lim_{x\to 0}\frac{\sin x}{x}}{ln\cdot\lim_{x\to 0}(1+x)^{\frac{1}{x}}}$$

$$\frac{3-1}{lne} = 2$$

$$\lim_{x\to 0}\frac{\sin 3x-\sin x}{In(1+x)} = 2$$

79.-$\lim\limits_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{\frac{1}{x}}$

$$\lim_{x\to 0}\left(\frac{a^0+b^0+c^0}{3}\right)^{\frac{1}{0}}=\left(\frac{3}{3}\right)^{\infty}=1^{\infty}$$

$$\lim_{x\to a}\left[\left(1+f_{(x)}\right)^{\frac{1}{f_{(x)}}}\right]^{f_{(x)}\cdot g_{(x)}}=e^{\lim\limits_{x\to a}f_{(x)}\cdot g_{(x)}}$$

$$\lim_{x\to 0}\left(1+\frac{a^x+b^x+c^x}{3}-1\right)^{\frac{1}{x}}=\lim_{x\to 0}\left(1+\frac{a^x+b^x+c^x-3}{3}\right)^{\frac{1}{x}}$$

$$\lim_{x\to 0}\left[\left(1+\frac{a^x+b^x+c^x-3}{3}\right)^{\frac{1}{\frac{a^x+b^x+c^x-3}{3}}}\right]^{\frac{a^x+b^x+c^x-3}{3}\cdot\frac{1}{x}}$$

$$e^{\lim\limits_{x\to 0}\frac{a^x+b^x+c^x-3}{3}\cdot\frac{1}{x}}$$

$$e^{\lim\limits_{x\to 0}\frac{(a^x-1)+(b^x-1)+(c^x-1)}{\mathrm{x}}\cdot\frac{1}{3}}=e^{\lim\limits_{x\to 0}\left(\frac{a^x-1}{x}+\frac{b^x-1}{x}+\frac{c^x-1}{x}\right)\cdot\frac{1}{3}}$$

$$e^{\frac{1}{3}\left(\lim\limits_{x\to 0}\left(\frac{a^x-1}{x}\right)+\lim\limits_{x\to 0}\left(\frac{b^x-1}{x}\right)+\lim\limits_{x\to 0}\left(\frac{c^x-1}{x}\right)\right)}$$

$$e^{\frac{1}{3}\left(\lim\limits_{x\to 0}\ln a+\lim\limits_{x\to 0}\ln b+\lim\limits_{x\to 0}\ln c\right)}$$

$$e^{\frac{1}{3}(\ln a+b+\ln c)}=e^{\frac{1}{3}(\ln abc)}=e^{\frac{\ln abc}{3}}$$

$$e^{\frac{1}{3}\cdot\ln abc} = e^{\ln abc^{\frac{1}{3}}} = e^{\ln \sqrt[3]{abc}}$$

$$\sqrt[3]{abc}^{\ln e} = \sqrt[3]{abc}$$

$$\lim_{x\to 0}\left(\frac{a^x + b^x + c^x}{3}\right)^{\frac{1}{x}} = \sqrt[3]{abc}$$

80.- $\lim_{x\to 0}\sqrt[x]{\frac{a^{x+1}+b^{x+1}+c^{x+1}}{a+b+c}}$

$$\lim_{x\to 0}\sqrt[0]{\frac{a^{0+1}+b^{0+1}+c^{0+1}}{a+b+c}}=1^{\infty}$$

$$\lim_{x\to 0}\sqrt[x]{\frac{a^{x+1}+b^{x+1}+c^{x+1}}{a+b+c}} = \lim_{x\to 0}\left(\frac{a^{x+1}+b^{x+1}+c^{x+1}}{a+b+c}\right)^{\frac{1}{x}}$$

$$\lim_{x\to 0}\left(1+\frac{a^{x+1}+b^{x+1}+c^{x+1}}{a+b+c}-1\right)^{\frac{1}{x}}$$

$$\lim_{x\to 0}\left(1+\frac{a^{x+1}+b^{x+1}+c^{x+1}-a-b-c}{a+b+c}\right)^{\frac{1}{x}}$$

$$\lim_{x\to 0}\left[\left(1+\frac{a^{x+1}+b^{x+1}+c^{x+1}-a-b-c}{a+b+c}\right)^{\frac{1}{\frac{a^{x+1}+b^{x+1}+c^{x+1}-a-b-c}{a+b+c}}}\right]^{\frac{a^{x+1}+b^{x+1}+c^{x+1}-a-b-c}{a+b+c}\cdot\frac{1}{x}}$$

$$e^{\lim\limits_{x\to 0}\frac{a^{x+1}+b^{x+1}+c^{x+1}-a-b-c}{a+b+c}\cdot\frac{1}{x}}$$

$$e^{\lim\limits_{x\to 0}\frac{a^{x+1}-a+b^{x+1}-b+c^{x+1}-c}{a+b+c}\cdot\frac{1}{x}}$$

$$e^{\lim\limits_{x\to 0}\frac{a(a^{x}-1)+b(b^{x}-1)+c(c^{x}-1)}{a+b+c}\cdot\frac{1}{x}}$$

$$e^{\lim\limits_{x\to 0}\frac{a(a^{x}-1)+b(b^{x}-1)+c(c^{x}-1)}{x}\cdot\frac{1}{a+b+c}}$$

$$e^{\lim\limits_{x\to 0}\left(\frac{a(a^{x}-1)}{x}+\frac{b(b^{x}-1)}{x}+\frac{c(c^{x}-1)}{x}\right)\cdot\frac{1}{a+b+c}}$$

$$e^{\left(\lim\limits_{x\to 0}\frac{a(a^{x}-1)}{x}+\lim\limits_{x\to 0}\frac{b(b^{x}-1)}{x}+\lim\limits_{x\to 0}\frac{c(c^{x}-1)}{x}\right)\cdot\frac{1}{a+b+c}}$$

$$e^{\left(\lim\limits_{x\to 0} a\cdot\ln a+\lim\limits_{x\to 0} b\cdot\ln b+\lim\limits_{x\to 0} c\cdot\ln c\right)\cdot\frac{1}{a+b+c}}$$

$$e^{(\ a\cdot\ln a+b\cdot\ln b+c\ln c)\cdot\frac{1}{a+b+c}}$$

$$e^{(a\cdot\ln a+b\cdot\ln b+c\ln c)\cdot\frac{1}{a+b+c}}$$

$$e^{\frac{(a\cdot\ln a+b\cdot\ln b+c\ln c)}{a+b+c}}$$

$$e^{\frac{a}{a+b+c}\cdot\ln a+\frac{b}{a+b+c}\cdot\ln b+\frac{c}{a+b+c}\cdot\ln c}$$

$$e^{\ln a^{\frac{a}{a+b+c}}+\ln b^{\frac{b}{a+b+c}}+\ln c^{\frac{c}{a+b+c}}}$$

$$e^{\ln a^{\frac{a}{a+b+c}}\cdot b^{\frac{b}{a+b+c}}\cdot c^{\frac{c}{a+b+c}}}$$

$$\left(a^{\frac{a}{a+b+c}}\cdot b^{\frac{b}{a+b+c}}\cdot c^{\frac{c}{a+b+c}}\right)^{\ln e}=(a^a\cdot b^b\cdot c^c)^{\frac{1}{a+b+c}}$$

81.- $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\ln(\cos x)}{x^2}$

$$\lim_{x\to\frac{\pi}{2}}\frac{\ln(\cos x)}{x^2}=\lim_{x\to\frac{\pi}{2}}\frac{1}{x^2}\ln\cos x$$

$$\lim_{x\to\frac{\pi}{2}}\ln\cos x^{\frac{1}{x^2}}=\ln\lim_{x\to\frac{\pi}{2}}\cos x^{\frac{1}{x^2}}$$

$$\lim_{x\to a}\left[\left(1+f_{(x)}\right)^{\frac{1}{f_{(x)}}}\right]^{f_{(x)}\cdot g_{(x)}}=e^{\lim\limits_{x\to a}f_{(x)}\cdot g_{(x)}}$$

$$\ln\lim_{x\to\frac{\pi}{2}}(1+\cos x-1)^{\frac{1}{x^2}}$$

$$\ln\lim_{x\to\frac{\pi}{2}}\left[(1+\cos x-1)^{\frac{1}{\cos x-1}}\right]^{\cos x-1\cdot\frac{1}{x^2}}$$

$$\ln e^{\lim\limits_{x\to\frac{\pi}{2}} \cos x - 1 \cdot \frac{1}{x^2}} = \ln e^{\lim\limits_{x\to\frac{\pi}{2}} -\frac{1-\cos x}{x^2}}$$

$$\ln e^{\lim\limits_{x\to\frac{\pi}{2}} -\frac{1}{2}} = \ln e^{-\frac{1}{2}} = -\frac{1}{2}\ln e = -\frac{1}{2}$$

$$\lim_{x\to\frac{\pi}{2}} \frac{\ln(\cos x)}{x^2} = -\frac{1}{2}$$

$$82. -\lim_{x\to 0} \frac{\ln(\cos ax)}{\ln(\cos bx)}$$

$$\lim_{x\to 0} \frac{\ln(\cos ax)}{\ln(\cos bx)} = \lim_{x\to 0} \frac{\frac{\ln(\cos ax)}{x^2}}{\frac{\ln(\cos bx)}{x^2}}$$

$$\lim_{x\to 0} \frac{\frac{1}{x^2}\ln(\cos ax)}{\frac{1}{x^2}\ln(\cos bx)} = \lim_{x\to 0} \frac{\ln(\cos ax)^{\frac{1}{x^2}}}{\ln(\cos bx)^{\frac{1}{x^2}}}$$

$$\frac{\lim\limits_{x\to 0} \ln(\cos ax)^{\frac{1}{x^2}}}{\lim\limits_{x\to 0} \ln(\cos bx)^{\frac{1}{x^2}}} = \frac{\ln \lim\limits_{x\to 0} (\cos ax)^{\frac{1}{x^2}}}{\ln \lim\limits_{x\to 0} (\cos bx)^{\frac{1}{x^2}}}$$

$$\lim_{x\to a}\left[\left(1+f_{(x)}\right)^{\frac{1}{f_{(x)}}}\right]^{f_{(x)}\cdot g_{(x)}} = e^{\lim\limits_{x\to a} f_{(x)}\cdot g_{(x)}}$$

$$\frac{\ln \lim\limits_{x\to 0}\left[(1+\cos ax-1)^{\frac{1}{\cos ax-1}}\right]^{\cos ax-1\cdot\frac{1}{x^2}}}{\ln \lim\limits_{x\to 0}\left[(1+\cos bx-1)^{\frac{1}{\cos bx-1}}\right]^{\cos bx-1\cdot\frac{1}{x^2}}}$$

$$\frac{\ln e^{\lim\limits_{x\to 0}\cos ax-1\cdot\frac{1}{x^2}}}{\ln e^{\lim\limits_{x\to 0}\cos bx-1\cdot\frac{1}{x^2}}} = \frac{\ln e^{\lim\limits_{x\to 0}\frac{\cos ax-1}{x^2}}}{\ln e^{\lim\limits_{x\to 0}\frac{\cos bx-1}{x^2}}}$$

$$\frac{\ln e^{\lim\limits_{x\to 0}\frac{\cos ax-1}{x^2}\cdot\frac{a^2}{a^2}}}{\ln e^{\lim\limits_{x\to 0}\frac{\cos bx-1}{x^2}\cdot\frac{b^2}{b^2}}} = \frac{\ln e^{\lim\limits_{x\to 0}\frac{\cos ax-1}{(ax)^2}\cdot\frac{a^2}{1}}}{\ln e^{\lim\limits_{x\to 0}\frac{\cos bx-1}{(bx)^2}\cdot\frac{b^2}{1}}}$$

$$\frac{\ln e^{a^2}}{\ln e^{b^2}} = \frac{a^2\cdot\ln e}{b^2\cdot\ln e} = \frac{a^2}{b^2}$$

$$\lim_{x\to 0}\frac{\ln(\cos ax)}{\ln(\cos bx)} = \frac{a^2}{b^2}$$

83.-$\lim_{x\to 0}\left[\sin\frac{1}{x}+\cos\frac{1}{x}\right]^{x}$

$$\lim_{x\to a}\left[\left(1+f_{(x)}\right)^{\frac{1}{f_{(x)}}}\right]^{f_{(x)}\cdot g_{(x)}}=e^{\lim\limits_{x\to a}f_{(x)}\cdot g_{(x)}}$$

$$\lim_{x\to 0}\left[\left(1+\sin\frac{1}{x}+\cos\frac{1}{x}-1\right)^{\frac{1}{\sin\frac{1}{x}+\cos\frac{1}{x}-1}}\right]^{(\sin\frac{1}{x}+\cos\frac{1}{x}-1)\cdot x}$$

$$e^{\lim\limits_{x\to 0}(\sin\frac{1}{x}+\cos\frac{1}{x}-1)\cdot x}=e^{\lim\limits_{x\to 0}\left(\sin\frac{1}{x}\right)\cdot x+(\cos\frac{1}{x}-1)\cdot x}$$

$$e^{\lim\limits_{x\to 0}\frac{\sin\frac{1}{x}}{\frac{1}{x}}-\frac{1-\cos\frac{1}{x}}{\frac{1}{x}}}=e^{\lim\limits_{x\to 0}1-0}=e$$

$$\lim_{x\to 0}\left[\sin\frac{1}{x}+\cos\frac{1}{x}\right]^{x}=e$$

84.-$\lim_{x\to a}\frac{x-a}{\ln x-Lna}$

$$\lim_{x\to a}\frac{x-a}{\ln x-Lna}=\lim_{x\to a}\frac{1}{(x-a)^{-1}}\cdot\frac{1}{\ln\frac{x}{a}}$$

$$\lim_{x\to a}\frac{1}{\ln\left(\frac{x}{a}\right)^{(x-a)^{-1}}}=\frac{1}{\ln\lim_{x\to a}\left(\frac{x}{a}\right)^{(x-a)^{-1}}}$$

$$\lim_{x\to a}\left[\left(1+f_{(x)}\right)^{\frac{1}{f_{(x)}}}\right]^{f_{(x)}\cdot g_{(x)}}=e^{\lim_{x\to a}f_{(x)}\cdot g_{(x)}}$$

$$\frac{1}{\ln\lim_{x\to a}\left(\frac{x}{a}\right)^{\frac{1}{(x-a)}}}=\frac{1}{\ln\lim_{x\to a}\left(1+\frac{x}{a}-1\right)^{\frac{1}{(x-a)}}}$$

$$\frac{1}{\ln\lim_{x\to a}\left(1+\frac{x-a}{a}\right)^{\frac{1}{(x-a)}}}=\frac{1}{\ln\lim_{x\to a}\left[\left(1+\frac{x-a}{a}\right)^{\frac{a}{(x-a)}}\right]^{\frac{1}{a}}}$$

$$\frac{1}{\ln e^{\lim_{x\to a}\frac{1}{a}}}=\frac{1}{\ln e^{\frac{1}{a}}}=\frac{1}{\frac{1}{a}\cdot\ln e}=a$$

$$\lim_{x\to a}\frac{x-a}{\ln x-Lna}=a$$

85. $-\lim_{x\to 0} \frac{1}{ax} \ln \sqrt[3]{\frac{(1+ax)}{(1-ax)}}$

$$\lim_{x\to 0} \ln \left(\sqrt[3]{\frac{(1+ax)}{(1-ax)}} \right)^{\frac{1}{ax}} = \ln \lim_{x\to 0} \left(\sqrt[3]{\frac{(1+ax)}{(1-ax)}} \right)^{\frac{1}{ax}}$$

$$\lim_{x\to a} \left[\left(1 + f_{(x)}\right)^{\frac{1}{f_{(x)}}} \right]^{f_{(x)} \cdot g_{(x)}} = e^{\lim_{x\to a} f_{(x)} \cdot g_{(x)}}$$

$$\ln \lim_{x\to 0} \left(1 + \sqrt[3]{\frac{(1+ax)}{(1-ax)}} - 1 \right)^{\frac{1}{ax}}$$

$$\ln \lim_{x\to 0} \left[\left(1 + \sqrt[3]{\frac{(1+ax)}{(1-ax)}} - 1 \right)^{\frac{1}{\sqrt[3]{\frac{(1+ax)}{(1-ax)}} - 1}} \right]^{\left(\sqrt[3]{\frac{(1+ax)}{(1-ax)}} - 1\right) \cdot \left(\frac{1}{ax}\right)}$$

$$\ln e^{\lim_{x\to 0} \left(\sqrt[3]{\frac{(1+ax)}{(1-ax)}} - 1\right) \cdot \left(\frac{1}{ax}\right)} = \ln e^{\lim_{x\to 0} \left(\frac{\sqrt[3]{1+ax} - \sqrt[3]{1-ax}}{\sqrt[3]{1-ax}}\right) \cdot \left(\frac{1}{ax}\right)}$$

$$\ln e^{\lim_{x\to 0}\left(\frac{\left(\sqrt[3]{1+ax}-\sqrt[3]{1-ax}\right)}{\sqrt[3]{1-ax}}\cdot\frac{\left(\left(\sqrt[3]{1+ax}\right)^2+\sqrt[3]{1-ax}\cdot\sqrt[3]{1+ax}+\left(\sqrt[3]{1-ax}\right)^2\right)}{\left(\left(\sqrt[3]{1+ax}\right)^2+\sqrt[3]{1-ax}\cdot\sqrt[3]{1+ax}+\left(\sqrt[3]{1-ax}\right)^2\right)}\right)\cdot\left(\frac{1}{ax}\right)}$$

$$\ln e^{\lim_{x\to 0}\left(\frac{\left(\sqrt[3]{1+ax}\right)^3-\left(\sqrt[3]{1-ax}\right)^3}{\sqrt[3]{1-ax}}\cdot\frac{1}{\left(\left(\sqrt[3]{1+ax}\right)^2+\sqrt[3]{1-ax}\cdot\sqrt[3]{1+ax}+\left(\sqrt[3]{1-ax}\right)^2\right)}\right)\cdot\left(\frac{1}{ax}\right)}$$

$$\ln e^{\lim_{x\to 0}\left(\frac{1+ax-1+ax}{\sqrt[3]{1-ax}}\cdot\frac{1}{\left(\left(\sqrt[3]{1+ax}\right)^2+\sqrt[3]{1-ax}\cdot\sqrt[3]{1+ax}+\left(\sqrt[3]{1-ax}\right)^2\right)}\right)\cdot\left(\frac{1}{ax}\right)}$$

$$\ln e^{\lim_{x\to 0}\left(\frac{2ax}{\sqrt[3]{1-ax}}\cdot\frac{1}{\left(\left(\sqrt[3]{1+ax}\right)^2+\sqrt[3]{1-ax}\cdot\sqrt[3]{1+ax}+\left(\sqrt[3]{1-ax}\right)^2\right)}\right)\cdot\left(\frac{1}{ax}\right)}$$

$$\ln e^{\lim_{x\to 0}\left(\frac{2}{\sqrt[3]{1-ax}}\cdot\frac{1}{\left(\left(\sqrt[3]{1+ax}\right)^2+\sqrt[3]{1-ax}\cdot\sqrt[3]{1+ax}+\left(\sqrt[3]{1-ax}\right)^2\right)}\right)}$$

$$\ln e^{\lim_{x\to 0}\left(\frac{2}{\sqrt[3]{1-a\cdot 0}}\cdot\frac{1}{\left(\left(\sqrt[3]{1+a\cdot 0}\right)^2+\sqrt[3]{1-a\cdot 0}\cdot\sqrt[3]{1+a\cdot 0}+\left(\sqrt[3]{1-a\cdot 0}\right)^2\right)}\right)}$$

$$\ln e^{\lim_{x\to 0}\left(\frac{2}{\sqrt[3]{1}}\cdot\frac{1}{\left(\left(\sqrt[3]{1}\right)^2+\sqrt[3]{1}\cdot\sqrt[3]{1}+\left(\sqrt[3]{1}\right)^2\right)}\right)}$$

$$\ln e^{\lim_{x\to 0}\left(\frac{2}{3}\right)} = \ln e^{\frac{2}{3}} = \frac{2}{3}\ln e = \frac{2}{3}$$

$$\lim_{x\to 0}\frac{1}{ax}\ln\sqrt[3]{\frac{(1+ax)}{(1-ax)}} = \frac{2}{3}$$

86.- $\lim_{h\to 0}\frac{\ln(x+h)-Lnx}{h}$

$$\lim_{h\to 0}\frac{\ln\frac{(x+h)}{x}}{h} = \lim_{h\to 0}\frac{1}{h}\ln\frac{(x+h)}{x}$$

$$\lim_{h\to 0}\ln\left(\frac{x+h}{x}\right)^{\frac{1}{h}} = \ln\lim_{h\to 0}\left(\frac{x+h}{x}\right)^{\frac{1}{h}}$$

$$\ln\lim_{h\to 0}\left(1+\frac{h}{x}\right)^{\frac{1}{h}} = \ln\lim_{h\to 0}\left[\left(1+\frac{h}{x}\right)^{\frac{x}{h}}\right]^{\frac{1}{x}}$$

$$\ln e^{\lim_{h\to a}\frac{1}{x}} = \ln e^{\frac{1}{x}} = \frac{1}{x}$$

$$\lim_{h\to 0}\frac{\ln(x+h)-Lnx}{h}=\frac{1}{x}$$

87.- $\lim_{x\to 0}\left(\sqrt{2-\sqrt{\cos x}}\right)^{\frac{1}{x^2}}$

$$\lim_{x\to a}\left[\left(1+f_{(x)}\right)^{\frac{1}{f_{(x)}}}\right]^{f_{(x)}\cdot g_{(x)}}=e^{\lim_{x\to a}f_{(x)}\cdot g_{(x)}}$$

$$\lim_{x\to 0}\left[\left(1+\sqrt{2-\sqrt{\cos x}}-1\right)^{\frac{1}{\sqrt{2-\sqrt{\cos x}}-1}}\right]^{\left(\sqrt{2-\sqrt{\cos x}}-1\right)\cdot\frac{1}{x^2}}$$

$$e^{\lim_{x\to 0}\left(\sqrt{2-\sqrt{\cos x}}-1\right)\cdot\frac{1}{x^2}}$$

$$e^{\lim_{x\to 0}\left(\sqrt{2-\sqrt{\cos x}}-1\right)\cdot\frac{\sqrt{2-\sqrt{\cos x}}+1}{\sqrt{2-\sqrt{\cos x}}+1}\cdot\frac{1}{x^2}}$$

$$e^{\lim_{x\to 0}\frac{\left(\sqrt{2-\sqrt{\cos x}}\right)^2-1^2}{\sqrt{2-\sqrt{\cos x}}+1}\cdot\frac{1}{x^2}}=e^{\lim_{x\to 0}\frac{2-\sqrt{\cos x}-1}{\sqrt{2-\sqrt{\cos x}}+1}\cdot\frac{1}{x^2}}$$

$$e^{\lim_{x\to 0}\frac{1-\sqrt{\cos x}}{\sqrt{2-\sqrt{\cos x}}+1}\cdot\frac{\left(1+\sqrt{\cos x}\right)}{\left(1+\sqrt{\cos x}\right)}\cdot\frac{1}{x^2}}$$

$$e^{\lim_{x\to 0}\frac{1-(\sqrt{\cos x})^2}{\sqrt{2-\sqrt{\cos x}}+1}\cdot\frac{1}{(1+\sqrt{\cos x})}\cdot\frac{1}{x^2}}$$

$$e^{\lim_{x\to 0}\frac{1-\cos x}{x^2}\cdot\frac{1}{(1+\sqrt{\cos x})}\cdot\frac{1}{\sqrt{2-\sqrt{\cos x}}+1}}$$

$$e^{\lim_{x\to 0}\frac{1}{2}\cdot\frac{1}{(1+\sqrt{\cos x})}\cdot\frac{1}{\sqrt{2-\sqrt{\cos x}}+1}}$$

$$e^{\lim_{x\to 0}\frac{1}{2}\cdot\frac{1}{(1+\sqrt{\cos 0})}\cdot\frac{1}{\sqrt{2-\sqrt{\cos 0}}+1}}$$

$$e^{\lim_{x\to 0}\frac{1}{2}\cdot\frac{1}{(2)}\cdot\frac{1}{2}} = e^{\lim_{x\to 0}\frac{1}{8}}$$

$$\lim_{x\to 0}\left(\sqrt{2-\sqrt{\cos x}}\right)^{\frac{1}{x^2}} = e^{\frac{1}{8}}$$

88.-$\lim_{x\to 0}\left(\frac{\cos x}{\cos 2x}\right)^{\frac{1}{x^2}}$

$$\lim_{x\to a}\left[\left(1+f_{(x)}\right)^{\frac{1}{f_{(x)}}}\right]^{f_{(x)}\cdot g_{(x)}} = e^{\lim_{x\to a} f_{(x)}\cdot g_{(x)}}$$

$$\lim_{x\to 0}\left[\left(1+\frac{\cos x-\cos 2x}{\cos 2x}\right)^{\frac{1}{\frac{\cos x-\cos 2x}{\cos 2x}}}\right]^{\frac{\cos x-\cos 2x}{\cos 2x}\cdot\frac{1}{x^2}}$$

$$e^{\lim_{x\to 0}\frac{\cos x-\cos 2x}{\cos 2x}\cdot\frac{1}{x^2}} = e^{\lim_{x\to 0}\frac{\cos x-\cos 2x}{x^2}\cdot\frac{1}{\cos 2x}}$$

$$e^{\lim_{x\to 0}\frac{\cos x-\left((\cos x)^2-(\sin x)^2\right)}{x^2}\cdot\frac{1}{\cos 2x}}$$

$$e^{\lim_{x\to 0}\frac{\cos x-(\cos x)^2+(\sin x)^2}{x^2}\cdot\frac{1}{\cos 2x}}$$

$$e^{\lim_{x\to 0}\frac{\cos x(1-\cos x)+(\sin x)^2}{x^2}\cdot\frac{1}{\cos 2x}}$$

$$e^{\lim_{x\to 0}\left(\frac{\cos x(1-\cos x)}{x^2}+\frac{(\sin x)^2}{x^2}\right)\cdot\frac{1}{\cos 2x}}$$

$$e^{\lim_{x\to 0}\left(\frac{\cos x}{2}+1\right)\cdot\frac{1}{\cos 2x}}$$

$$e^{\lim_{x\to 0}\left(\frac{\cos 0}{2}+1\right)\cdot\frac{1}{\cos 2\cdot 0}} = e^{\lim_{x\to 0}\left(\frac{1}{2}+1\right)}$$

$$e^{\left(\frac{1}{2}+1\right)} = e^{\frac{3}{2}}$$

$$\lim_{x\to 0}\left(\frac{\cos x}{\cos 2x}\right)^{\frac{1}{x^2}} = e^{\frac{3}{2}}$$

89.- $\lim_{x\to 0}\left(\frac{1+\tan x}{1-\sin x}\right)^{\frac{1}{\sin x}}$

$$\lim_{x\to a}\left[\left(1+f_{(x)}\right)^{\frac{1}{f_{(x)}}}\right]^{f_{(x)}\cdot g_{(x)}} = e^{\lim_{x\to a} f_{(x)}\cdot g_{(x)}}$$

$$\lim_{x\to 0}\left(1+\frac{1+\tan x}{1-\sin x}-1\right)^{\frac{1}{\sin x}}$$

$$\lim_{x\to 0}\left[\left(1+\frac{\tan x+\sin x}{1-\sin x}\right)^{\frac{1}{\frac{\tan x+\sin x}{1-\sin x}}}\right]^{\frac{\tan x+\sin x}{1-\sin x}\cdot\frac{1}{\sin x}}$$

$$e^{\lim_{x\to 0}\frac{\tan x+\sin x}{1-\sin x}\cdot\frac{1}{\sin x}} = e^{\lim_{x\to 0}\frac{\frac{\sin x}{\cos x}+\sin x}{1-\sin x}\cdot\frac{1}{\sin x}}$$

$$e^{\lim_{x\to 0}\frac{\frac{\sin x+\sin x\cdot\cos x}{\cos x}}{1-\sin x}\cdot\frac{1}{\sin x}} = e^{\lim_{x\to 0}\frac{\sin x+\sin x\cdot\cos x}{(\cos x)(1-\sin x)}\cdot\frac{1}{\sin x}}$$

$$e^{\lim_{x\to 0}\frac{\sin x(1+\cos x)}{(\cos x)(1-\sin x)}\cdot\frac{1}{\sin x}} = e^{\lim_{x\to 0}\frac{1+\cos x}{(\cos x)(1-\sin x)}}$$

$$e^{\lim_{x\to 0}\frac{1+\cos x}{(\cos x)(1-\sin x)}\cdot\frac{(1+\sin x)}{(1+\sin x)}}$$

$$e^{\lim\limits_{x\to 0}\frac{1+\cos x}{(\cos x)(1-(\sin x)^2)}\cdot\frac{(1+\sin x)}{1}}$$

$$e^{\lim\limits_{x\to 0}\frac{1+\cos x}{(\cos x)\cdot(\cos x)^2}\cdot\frac{(1+\sin x)}{1}} = e^{\lim\limits_{x\to 0}\frac{1+\cos x}{(\cos x)^3}\cdot\frac{(1+\sin x)}{1}}$$

$$e^{\lim\limits_{x\to 0}\frac{1+1}{(\cos 0)^3}\cdot\frac{(1+\sin 0)}{1}} = e^{\lim\limits_{x\to 0}\frac{2}{1}\cdot\frac{(1+0)}{1}} = e^2$$

$$\lim_{x\to 0}\left(\frac{1+\tan x}{1-\sin x}\right)^{\frac{1}{\sin x}} = e^2$$

90.- $\lim_{x \to 0}(\cos x + a\sin bx)^{\frac{1}{x}}$

$$\lim_{x \to a}\left[\left(1 + f_{(x)}\right)^{\frac{1}{f(x)}}\right]^{f(x) \cdot g(x)} = e^{\lim_{x \to a} f(x) \cdot g(x)}$$

$$\lim_{x \to 0}\left[(1 + \cos x + a\sin bx - 1)^{\frac{1}{\cos x + a\sin bx - 1}}\right]^{(\cos x + a\sin bx - 1) \cdot \frac{1}{x}}$$

$$e^{\lim_{x \to 0}(\cos x + a\sin bx - 1) \cdot \frac{1}{x}} = e^{\lim_{x \to 0}(\cos x - 1 + a\sin bx) \cdot \frac{1}{x}}$$

$$e^{\lim_{x \to 0}\left(\frac{\cos x - 1}{x} + \frac{a\sin bx}{x}\right)} = e^{\lim_{x \to 0}\left(-\frac{1 - \cos x}{x} + \frac{a\sin bx}{bx} \cdot \frac{b}{1}\right)}$$

$$e^{\lim_{x \to 0}\left(-\frac{1 - \cos x}{x} + \frac{a\sin bx}{bx} \cdot \frac{b}{1}\right)} = e^{\lim_{x \to 0}\left(-0 + \frac{ab}{1}\right)} = e^{ab}$$

$$\lim_{x \to 0}(\cos x + a\sin bx)^{\frac{1}{x}} = e^{ab}$$

91.- $\lim_{x \to 0} \sqrt[x]{1 - 2x}$

$$\lim_{x \to 0}(1 - 2x)^{\frac{1}{x}}$$

$$\lim_{x \to a}\left[\left(1 + f_{(x)}\right)^{\frac{1}{f(x)}}\right]^{f(x) \cdot g(x)} = e^{\lim_{x \to a} f(x) \cdot g(x)}$$

$$\lim_{x\to 0}\left[\left(1+(-2x)\right)^{\frac{1}{-2x}}\right]^{-2} = e^{\lim\limits_{x\to 0} -2} = e^{-2}$$

$$\lim_{x\to 0}\sqrt[x]{1-2x} = -2$$

92.- $\lim\limits_{x\to\infty}\left(\frac{x+a}{x-a}\right)^{x}$

$$\lim_{x\to a}\left[\left(1+f_{(x)}\right)^{\frac{1}{f_{(x)}}}\right]^{f_{(x)}\cdot g_{(x)}} = e^{\lim\limits_{x\to a} f_{(x)}\cdot g_{(x)}}$$

$$\lim_{x\to\infty}\left(1+\frac{x+a}{x-a}-1\right)^{x}$$

$$\lim_{x\to\infty}\left(1+\frac{x+a-x+a}{x-a}\right)^{x} = \lim_{x\to\infty}\left(1+\frac{2a}{x-a}\right)^{x}$$

$$\lim_{x\to\infty}\left[\left(1+\frac{2a}{x-a}\right)^{\frac{1}{\frac{2a}{x-a}}}\right]^{\frac{2a}{x-a}\cdot \mathrm{x}} = e^{\lim\limits_{x\to\infty}\frac{2a}{x-a}\cdot \mathrm{x}}$$

$$e^{\lim\limits_{x\to\infty}\frac{2ax}{x-a}} = e^{\lim\limits_{x\to\infty}\frac{\frac{2ax}{x}}{\frac{x-a}{x}}} = e^{\lim\limits_{x\to\infty}\frac{2a}{\frac{x}{x}-\frac{a}{x}}}$$

$$e^{\lim\limits_{x\to\infty}\frac{2a}{1-\frac{a}{\infty}}} = e^{\lim\limits_{x\to\infty}\frac{2a}{1-0}} = e^{2a}$$

$$\lim_{x\to\infty}\left(\frac{x+a}{x-a}\right)^{x}=e^{2a}$$

93.- $\lim_{x\to\infty}\left(\frac{x^2-1}{x^2+1}\right)^{x^2}$

$$\lim_{x\to a}\left[\left(1+f_{(x)}\right)^{\frac{1}{f_{(x)}}}\right]^{f_{(x)}\cdot g_{(x)}}=e^{\lim_{x\to a}f_{(x)}\cdot g_{(x)}}$$

$$\lim_{x\to\infty}\left(1+\frac{x^2-1}{x^2+1}-1\right)^{x^2}=\lim_{x\to\infty}\left(1+\frac{-2}{x^2+1}\right)^{x^2}$$

$$\lim_{x\to\infty}\left[\left(1+\frac{-2}{x^2+1}\right)^{\frac{1}{\frac{-2}{x^2+1}}}\right]^{\frac{-2}{x^2+1}\cdot x^2}=e^{\lim_{x\to\infty}\frac{-2}{x^2+1}\cdot x^2}$$

$$e^{\lim_{x\to\infty}\frac{-2x^2}{x^2+1}}=e^{\lim_{x\to\infty}-\frac{\frac{2x^2}{x^2}}{\frac{x^2+1}{x^2}}}=e^{\lim_{x\to\infty}-\frac{2}{\frac{x^2}{x^2}+\frac{1}{x^2}}}$$

$$e^{\lim_{x\to\infty}-\frac{2}{1+\frac{1}{x^2}}}=e^{-2}$$

$$\lim_{x\to\infty}\left(\frac{x^2-1}{x^2+1}\right)^{x^2}=e^{-2}$$

94.- $\lim_{x\to 0} \frac{Ln(1+x+x^2)-\text{Ln}(1-x+x^2)}{x^2}$

$$\lim_{x\to 0} \frac{1}{x^2} \cdot \frac{Ln(1+x+x^2)-\text{Ln}(1-x+x^2)}{1}$$

$$\lim_{x\to 0} \frac{1}{x^2} \cdot Ln\frac{(1+x+x^2)}{(1-x+x^2)} = \lim_{x\to 0} Ln\left(\frac{(1+x+x^2)}{(1-x+x^2)}\right)^{\frac{1}{x^2}}$$

$$Ln \lim_{x\to 0}\left(1+\frac{(1+x+x^2)}{(1-x+x^2)}-1\right)^{\frac{1}{x^2}}$$

$$Ln \lim_{x\to 0}\left(1+\frac{1+x+x^2-1-x-x^2}{(1-x+x^2)}\right)^{\frac{1}{x^2}}$$

$$\lim_{x\to a}\left[\left(1+f_{(x)}\right)^{\frac{1}{f_{(x)}}}\right]^{f_{(x)}\cdot g_{(x)}} = e^{\lim\limits_{x\to a} f_{(x)}\cdot g_{(x)}}$$

$$\lim_{x\to 0}\left[\left(1+\frac{2x}{1-x+x^2}\right)^{\frac{1}{\frac{2x}{1-x+x^2}}}\right]^{\frac{2x}{1-x+x^2}\cdot\frac{1}{x^2}}$$

$$e^{\lim\limits_{x\to 0}\frac{2x}{1-x+x^2}\cdot\frac{1}{x^2}}$$

$$e^{\lim\limits_{x\to 0}\frac{2}{1-x+x^2}\cdot\frac{1}{x}}$$

$$e^{\lim\limits_{x\to 0}\frac{2}{1+x^2-x}\cdot\frac{1+x^2+x}{1+x^2+x}\cdot\frac{1}{x}}$$

$$e^{\lim\limits_{x\to 0}\frac{2}{1+x^2+x^4}\cdot\frac{1+x^2+x}{1}\cdot\frac{1}{x}}$$

$$e^{\lim\limits_{x\to 0}\frac{2}{1+x^2+x^4}\cdot\left(\frac{1}{x}+\frac{x^2}{x}+\frac{x}{x}\right)}$$

$$e^{\lim\limits_{x\to 0}\frac{2}{1+x^2+x^4}\cdot\left(\frac{1}{x}+x+1\right)}$$

$$e^{\lim\limits_{x\to 0}\frac{2}{1+1+1}} = e^{\frac{2}{3}}$$

$$\lim_{x\to 0}\frac{Ln(1+x+x^2)-\mathrm{Ln}(1-x+x^2)}{x^2} = e^{\frac{2}{3}}$$

95. $-\lim_{x\to 0}\left(\sqrt[\sqrt{3}]{1+\sin\sqrt{3}\cdot x}\right)^{\frac{1}{\sin\sqrt{3}\cdot x}}$

$$\lim_{x\to 0}\left(1+\sin\sqrt{3}\cdot x\right)^{\frac{1}{\sqrt{3}\sin\sqrt{3}\cdot x}}$$

$$\lim_{x\to 0}\left[\left(1+\sin\sqrt{3}\cdot x\right)^{\frac{1}{\sin\sqrt{3}\cdot x}}\right]^{\sqrt{3}}$$

$$e^{\lim\limits_{x\to 0}\sqrt{3}}=e^{\sqrt{3}}$$

$$\lim_{x\to 0}\left(\sqrt[\sqrt{3}]{1+\sin\sqrt{3}\cdot x}\right)^{\frac{1}{\sin\sqrt{3}\cdot x}}=e^{\sqrt{3}}$$

96.- $\lim_{x\to a}\frac{x-a}{\ln x-Lna}$

$$\lim_{x\to a}\frac{x-a}{\ln x-Lna}=\lim_{x\to a}\frac{1}{(x-a)^{-1}}\cdot\frac{1}{\ln\frac{x}{a}}$$

$$\lim_{x\to a}\frac{1}{\ln\left(\frac{x}{a}\right)^{(x-a)^{-1}}}=\frac{1}{\ln\lim\limits_{x\to a}\left(\frac{x}{a}\right)^{(x-a)^{-1}}}$$

$$\lim_{x\to a}\left[\left(1+f_{(x)}\right)^{\frac{1}{f_{(x)}}}\right]^{f_{(x)}\cdot g_{(x)}} = e^{\lim\limits_{x\to a} f_{(x)}\cdot g_{(x)}}$$

$$\frac{1}{\ln \lim\limits_{x\to a}\left(\frac{x}{a}\right)^{\frac{1}{(x-a)}}} = \frac{1}{\ln \lim\limits_{x\to a}\left(1+\frac{x}{a}-1\right)^{\frac{1}{(x-a)}}}$$

$$\frac{1}{\ln \lim\limits_{x\to a}\left(1+\frac{x-a}{a}\right)^{\frac{1}{(x-a)}}} = \frac{1}{\ln \lim\limits_{x\to a}\left[\left(1+\frac{x-a}{a}\right)^{\frac{a}{(x-a)}}\right]^{\frac{1}{a}}}$$

$$\frac{1}{\ln e^{\lim\limits_{x\to a}\frac{1}{a}}} = \frac{1}{\ln e^{\frac{1}{a}}} = \frac{1}{\frac{1}{a}\cdot \ln e} = a$$

$$\lim_{x\to a}\frac{x-a}{\ln x - Lna} = a$$

97.- $\lim\limits_{h\to 0}\frac{a^{x+h}+a^{x-h}-2a^{x}}{h^2}$

$$\lim_{h\to 0}\frac{a^{x}\left(a^{h}+a^{-h}-2\right)}{h^2}$$

$$\lim_{h\to 0}\frac{a^{x}\left(a^{h}+\frac{1}{a^{h}}-2\right)}{h^2}$$

$$\lim_{h\to 0}\frac{a^x\left(\dfrac{a^{2h}+1-2a^h}{a^h}\right)}{h^2}$$

$$\lim_{h\to 0}\frac{a^x\left(\dfrac{a^{2h}-2a^h+1}{a^h}\right)}{h^2}$$

$$\lim_{h\to 0}\frac{a^x\cdot\left(a^h-1\right)^2}{a^h\cdot h^2}$$

$$\lim_{h\to 0}\frac{a^x}{a^h}\cdot\lim_{h\to 0}\left(\frac{a^h-1}{h}\right)^2$$

$$\lim_{h\to 0}\frac{a^x}{a^h}\cdot(\ln a)^2=\lim_{h\to 0}a^{x-h}\cdot(\ln a)^2=a^x\cdot(\ln a)^2$$

$$\lim_{h\to 0}\frac{a^{x+h}+a^{x-h}-2a^x}{h^2}=a^x\cdot(\ln a)^2$$

98.- $\lim_{x\to 0}\frac{e^{ax}-1}{b^{bx}-1}$

$$\lim_{x\to 0}\frac{\frac{e^{ax}-1}{ax}\cdot\frac{ax}{1}}{\frac{b^{bx}-1}{bx}\cdot\frac{bx}{1}}$$

$$\lim_{x\to 0}\frac{\ln e\cdot\frac{ax}{1}}{\ln b\cdot\frac{bx}{1}}=\lim_{x\to 0}\frac{ax}{\ln b\cdot bx}=\frac{a}{\ln b\cdot b}$$

$$\lim_{x\to 0}\frac{e^{ax}-1}{b^{bx}-1}=\frac{a}{b\ ln\ b}$$

99.- $\lim_{x\to 0}\frac{5^x-4^x}{x^2-x}$

$$\lim_{x\to 0}\frac{(5^x-1)-(4^x-1)}{x(x-1)}$$

$$\lim_{x\to 0}\frac{(5^x-1)-(4^x-1)}{x}\cdot\frac{1}{(x-1)}$$

$$\lim_{x\to 0}\left(\frac{5^x-1}{x}-\frac{4^x-1}{x}\right)\frac{1}{(x-1)}$$

$$\lim_{x\to 0}(\ln 5-\ln 4)\frac{1}{(x-1)}$$

$$\lim_{x\to 0}\left(\ln\frac{5}{4}\right)\frac{1}{(0-1)}=-1\lim_{x\to 0}\ln\frac{5}{4}$$

$$\lim_{x\to 0}\frac{5^x-4^x}{x^2-x}=\ln\frac{5}{4}$$

100.-$\lim_{x\to 0}\frac{\sin^2 3x}{\ln^2(1+2x)}$

$$\lim_{x\to 0}\left(\frac{sin\,3x}{\ln(1+2x)}\right)^2$$

$$\lim_{x\to 0}\left(\frac{\frac{sin\,3x}{3x}}{\frac{\ln(1+2x)}{3x}}\right)^2=\lim_{x\to 0}\left(\frac{1}{\frac{1}{3x}\cdot\ln(1+2x)}\right)^2$$

$$\lim_{x\to 0}\left(\frac{1}{\ln(1+2x)^{\frac{1}{3x}}}\right)^2=\frac{1}{\left\{\ln\lim_{x\to 0}(1+2x)^{\frac{1}{3x}}\right\}^2}$$

$$\frac{1}{\left\{\ln\lim_{x\to 0}\left[(1+2x)^{\frac{1}{2x}}\right]^{\frac{2}{3}}\right\}^2}=\frac{1}{\left\{\ln e^{\frac{2}{3}}\right\}^2}=\frac{1}{\left\{\frac{2}{3}\right\}^2}=\frac{1}{\frac{4}{9}}=\frac{9}{4}$$

$$\lim_{x\to 0}\frac{\sin^2 3x}{\ln^2(1+2x)}=\frac{9}{4}$$

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